Polar Coded Merkle Tree: Improved Detection of Data Availability Attacks in Blockchain Systems

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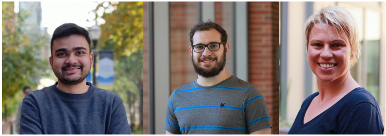
ISIT 2022



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Blockchain

- Distributed Ledger
- Decentralized trust platforms

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- Main Application:
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- Emerging Applications:
  - Healthcare services
  - Supply chain management
  - Industrial IoT
  - e-voting

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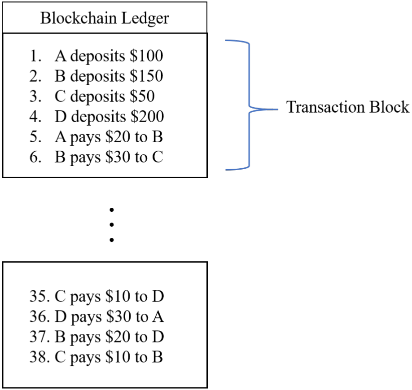

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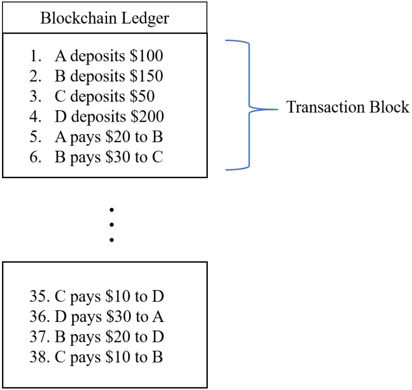

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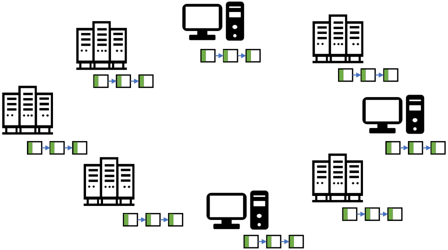

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- Stored by a network of nodes
- Full nodes: store a copy of the entire ledger

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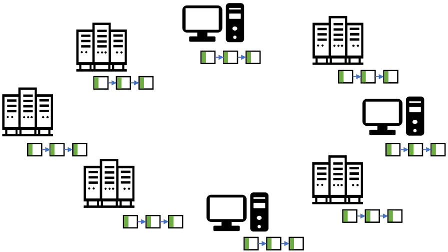

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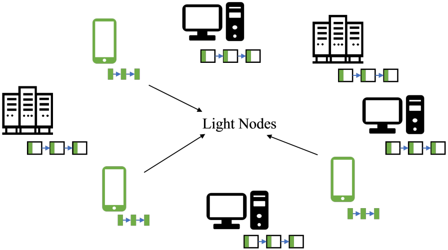
- Bitcoin ledger size $\sim$ 400GB[1]
- Ethereum ledger size $\sim$ 730GB[2]

As of 6/5/2022, [1]https://www.blockchain.com/charts/blocks-size
[2]https://etherscan.io/chartsync/chaindefault

Blockchain



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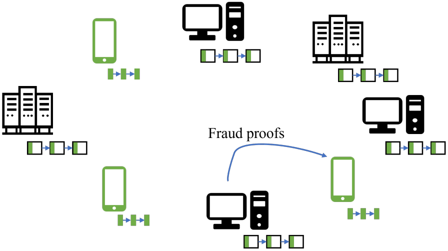

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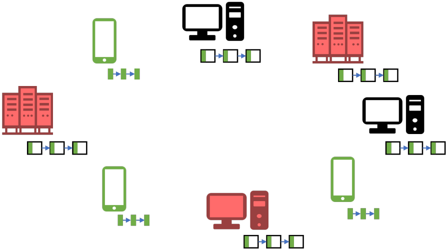

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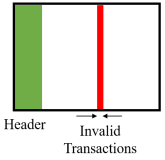
Systems with light nodes and a dishonest majority of full nodes are vulnerable to data availability attacks [Al-Bassam '18], [Yu '19]

Data Availability (DA) Attack

Adversary creates an invalid block



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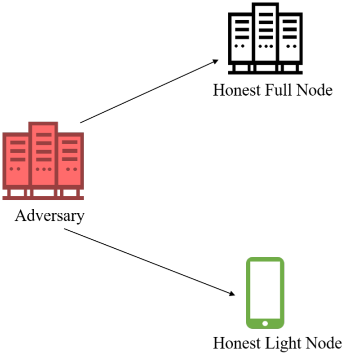

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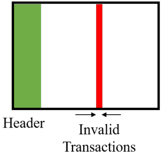

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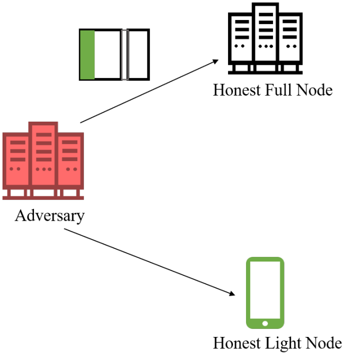

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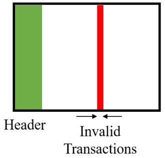

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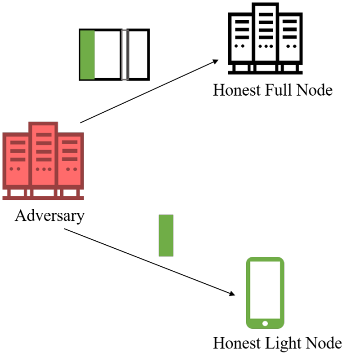

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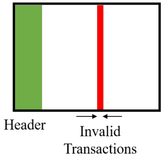

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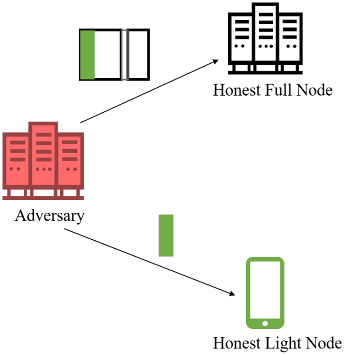

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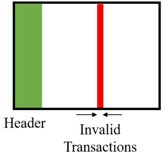

- Adversary: Provides block to Full node but hides invalid portion
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- Honest Nodes: Cannot verify missing transactions

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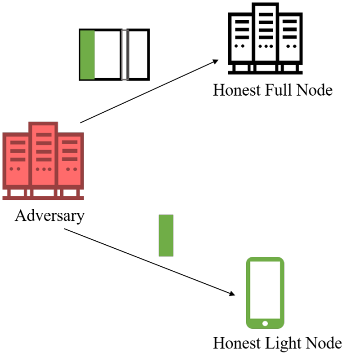

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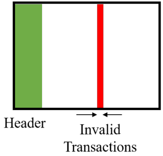

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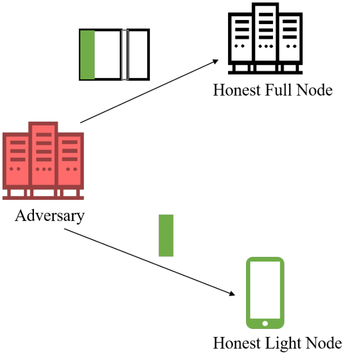

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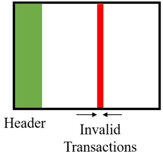

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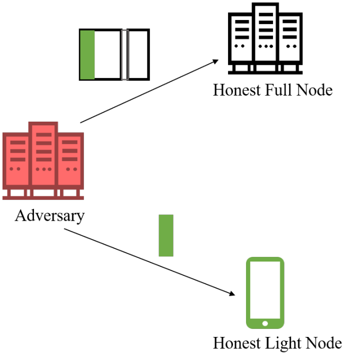

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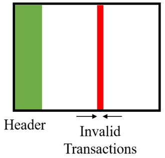

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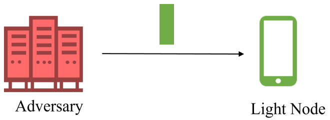
Solution: Light Node Sampling + Merkle Trees



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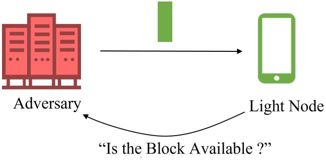

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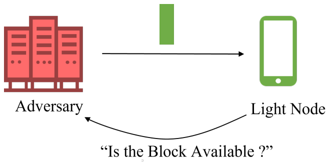

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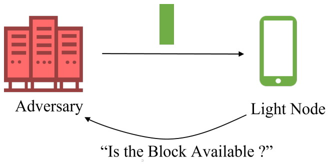

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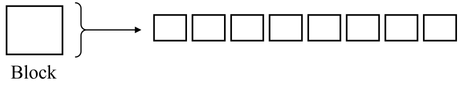

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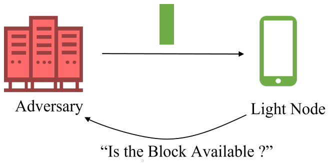

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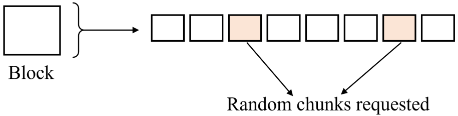

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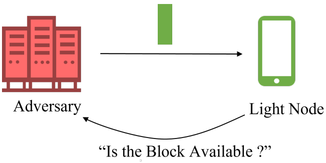

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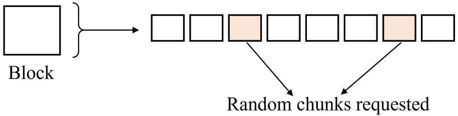

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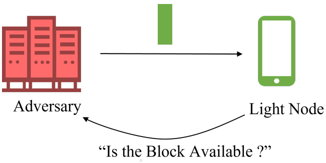

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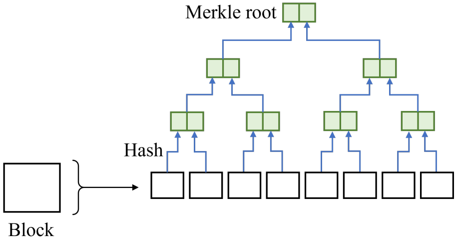

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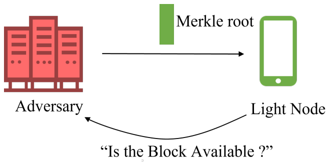

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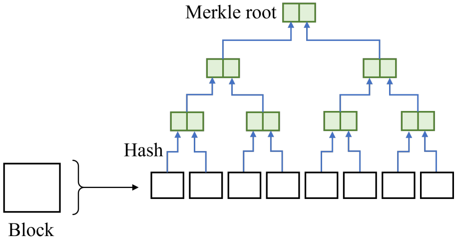

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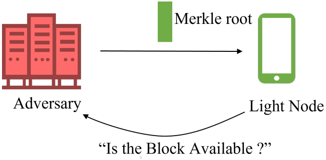

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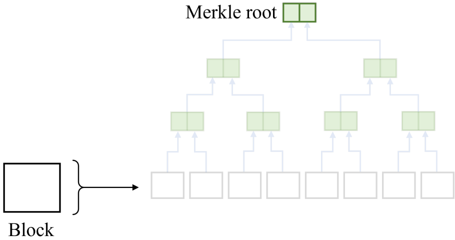

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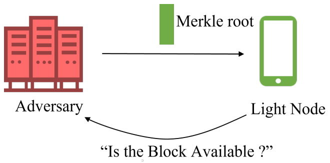

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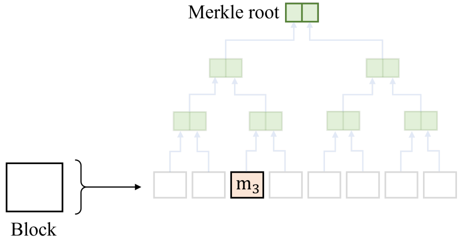

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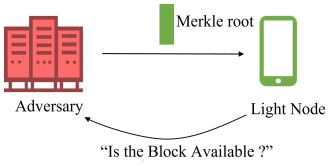

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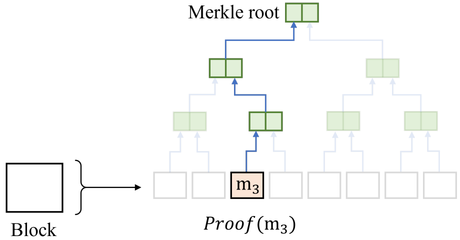

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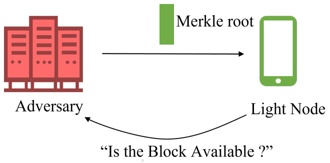

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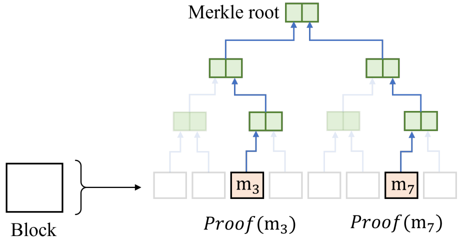

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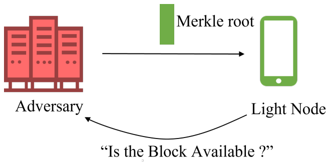

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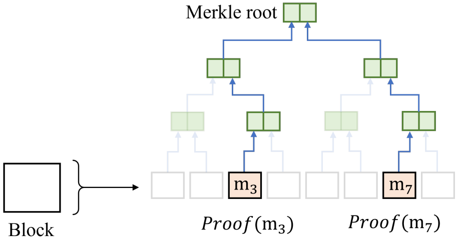

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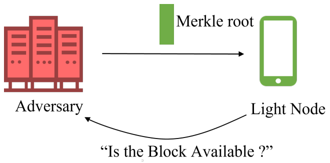

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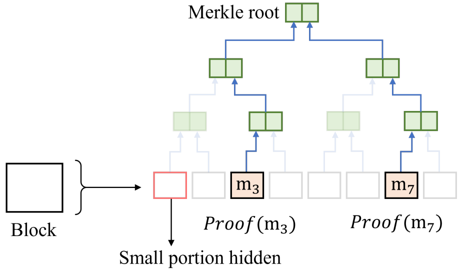

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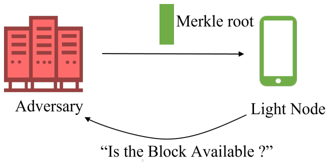

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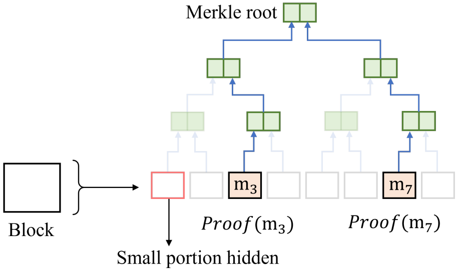

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Probability of failure using 2 random samples:

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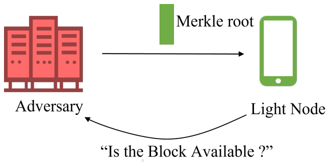

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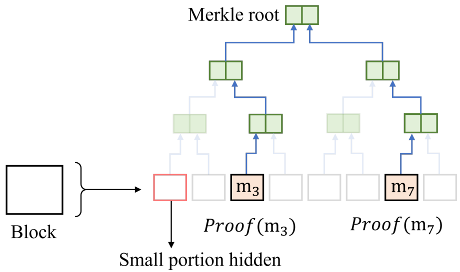

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Probability of failure using 2 random samples:

$$\left(1-\frac{1}{8}\right)\left(1-\frac{1}{7}\right) = 0.75$$

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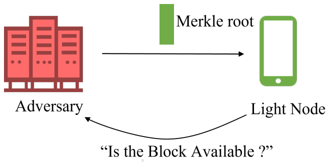

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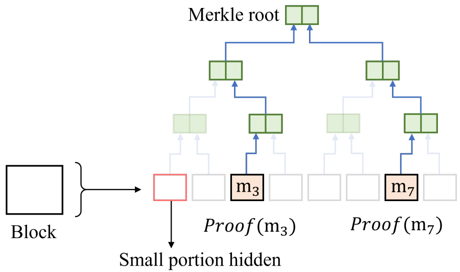

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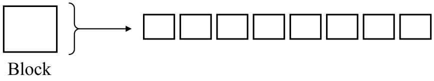
Erasure coding is used to improve the probability of failure

Erasure coding to Improve the Probability of Failure



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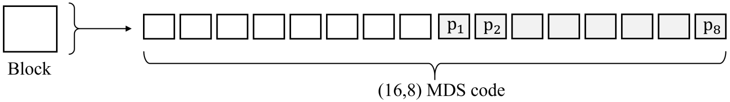

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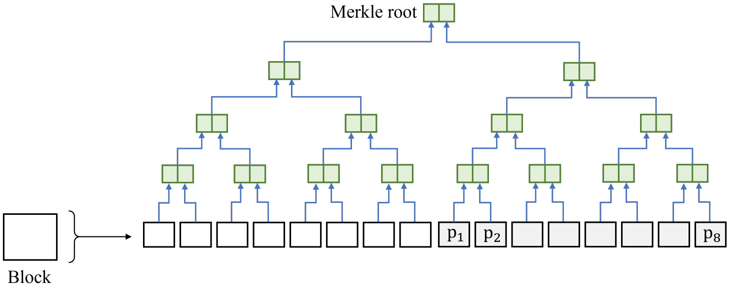

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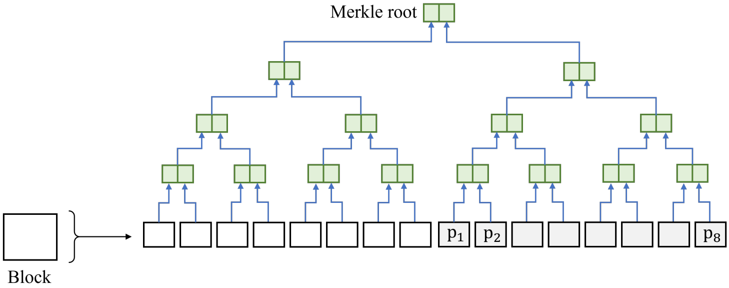

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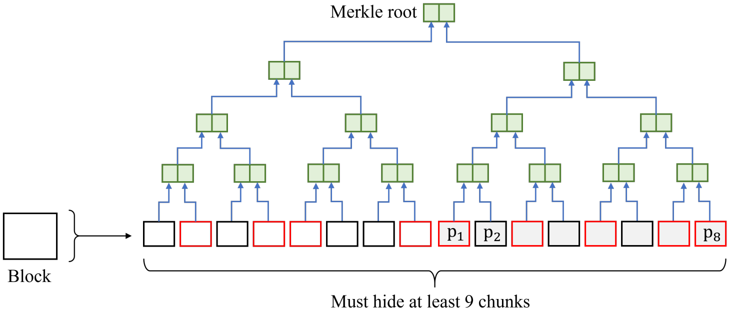

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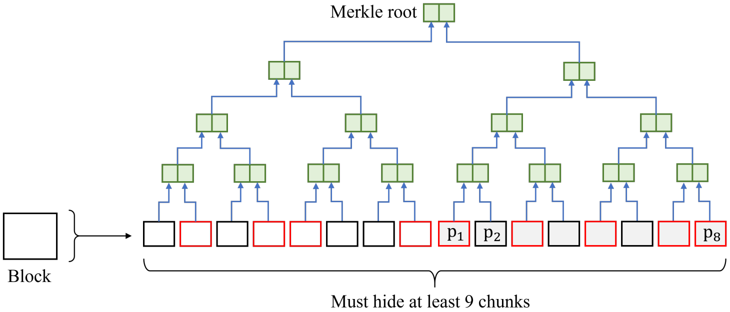
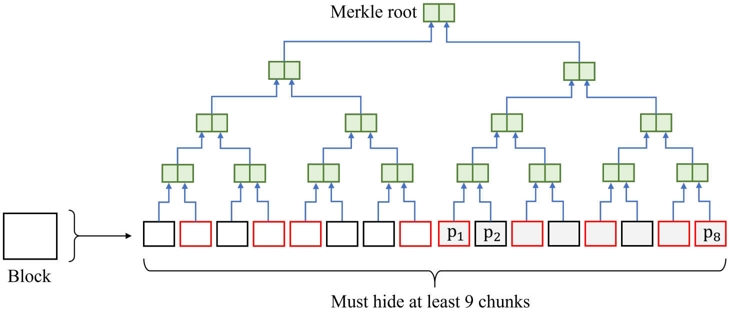


- Adversary must hide more coded chunks $\rightarrow$ easier for light nodes to catch using random sampling

Erasure coding to Improve the Probability of Failure



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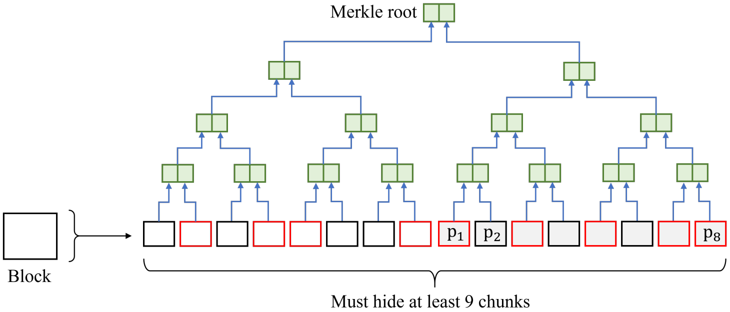

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Adversary can incorrectly encode the block!

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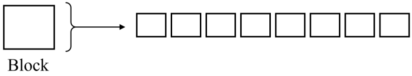

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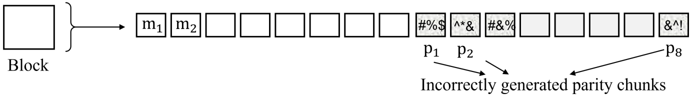

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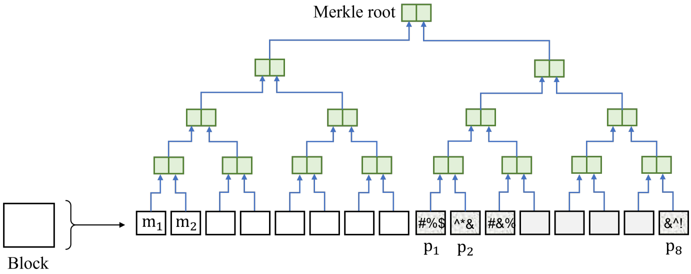

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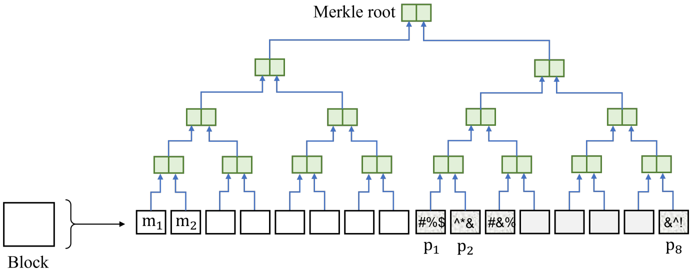

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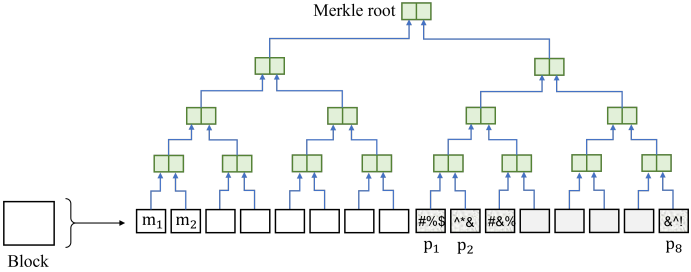

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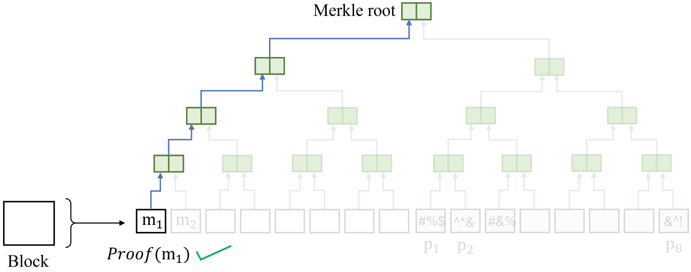

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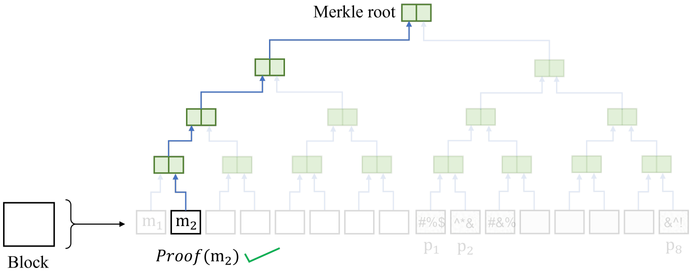

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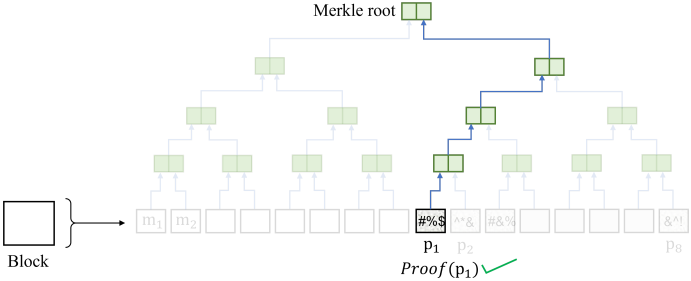

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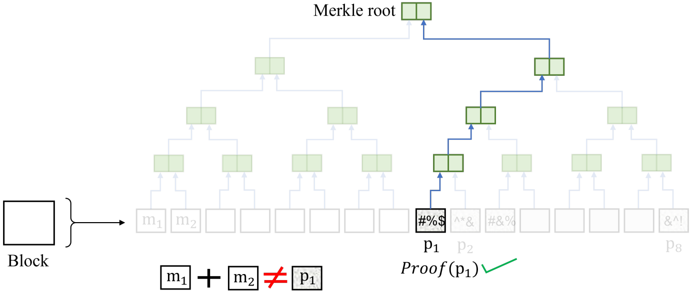

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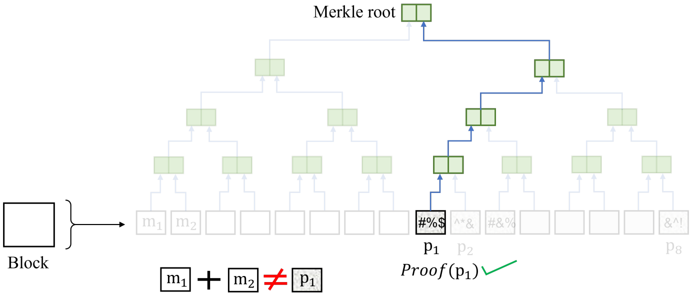

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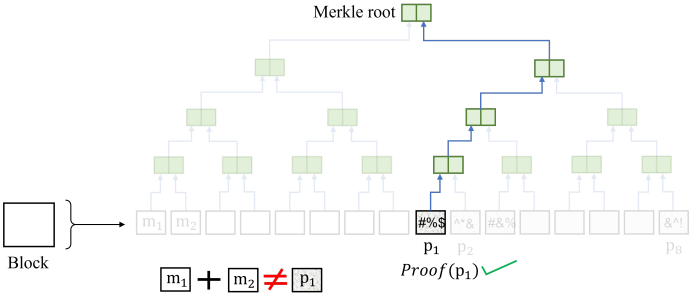

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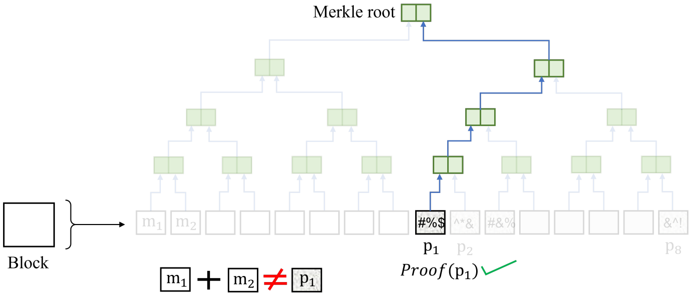

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IC-Proof size- 1D-RS: $O(b)$, 2D-RS [Al-Bassam '18] [Santini '22]: $O(\sqrt{b})$

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Our work: A novel construction of Merkle trees using polar codes that performs well on all the above metrics for large transaction block sizes.

Coded Merkle Tree (CMT) [Yu '19]

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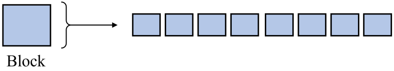

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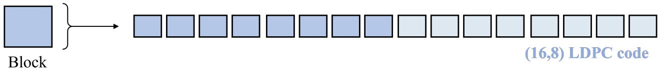

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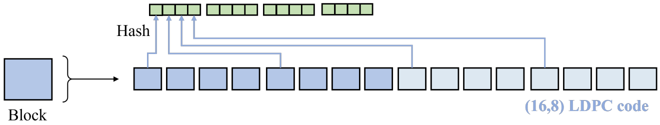

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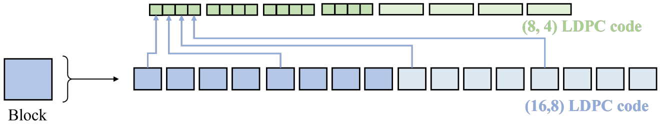

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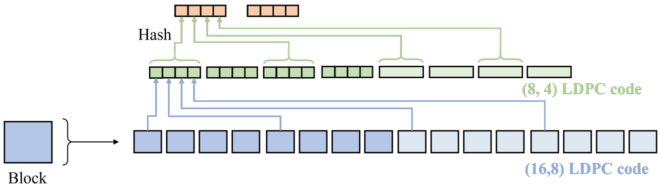

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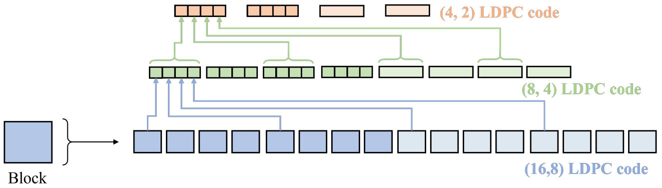

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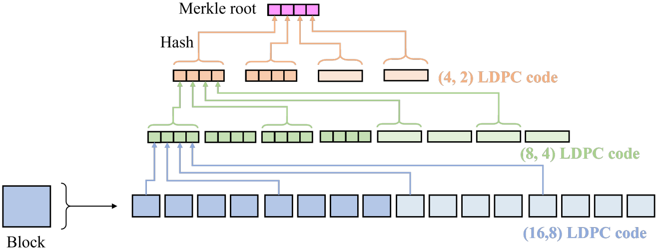

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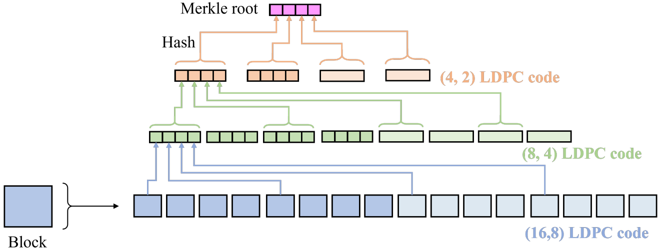

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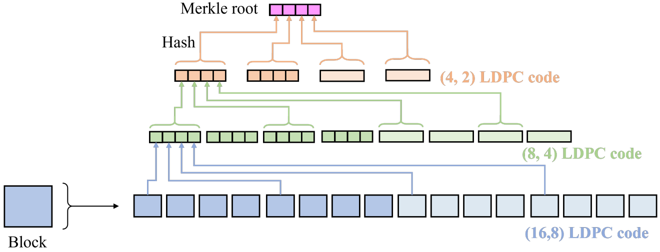

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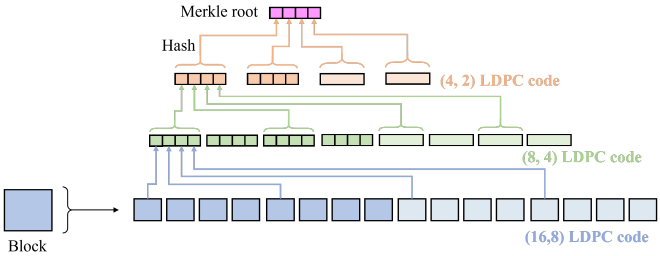

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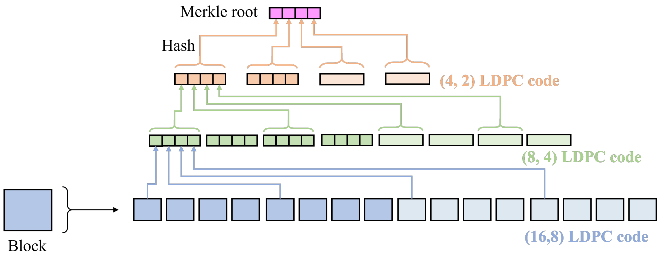

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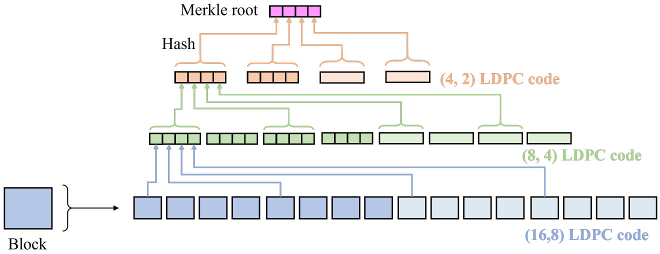

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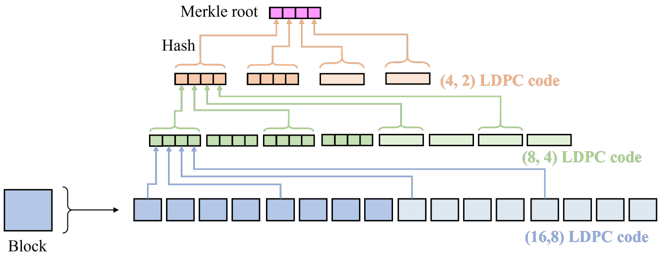

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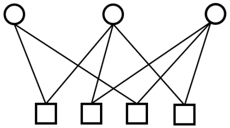

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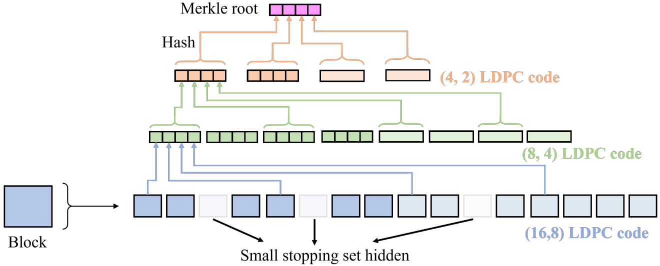

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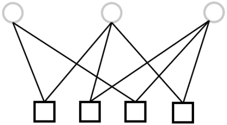

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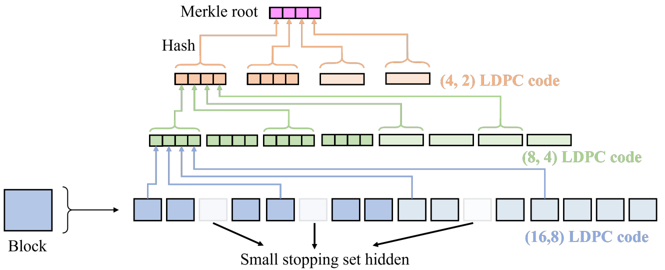

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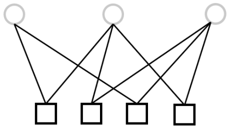

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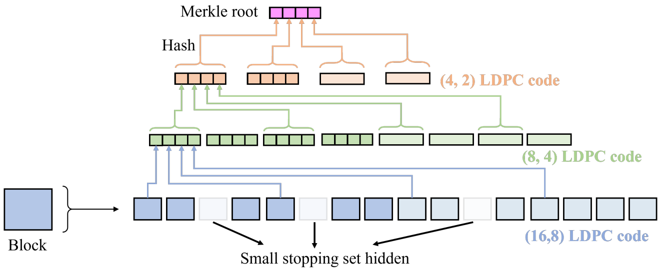

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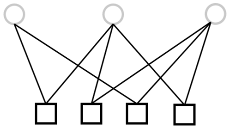

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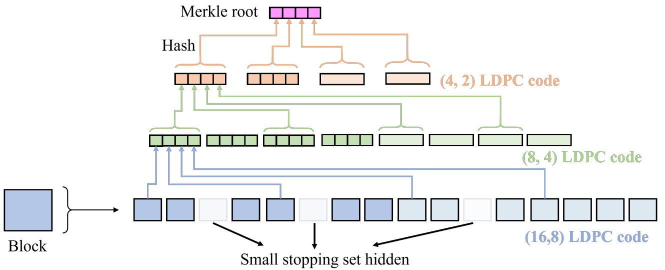

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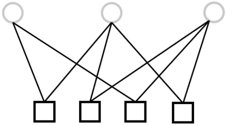

Merkle tree construction using polar codes allows for an efficient method to compute $\alpha_{\min}$

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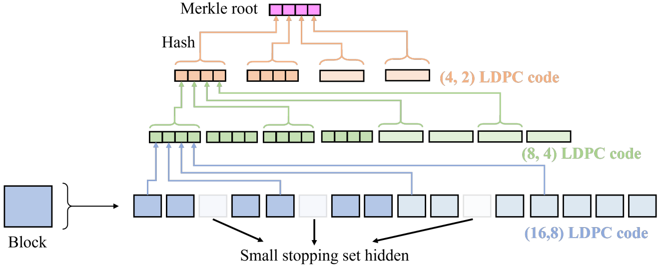

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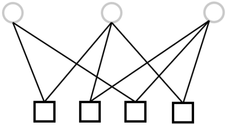

Merkle tree construction using polar codes allows for an efficient method to compute $\alpha_{\min}$ while having small IC-proof size and decoding complexity.

Polar Coded Merkle Tree (PCMT)

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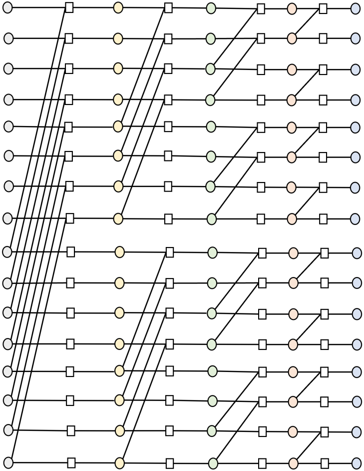

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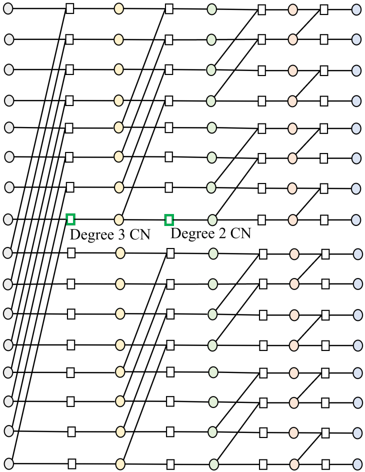

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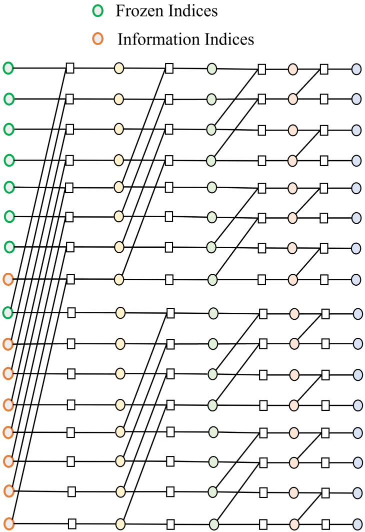

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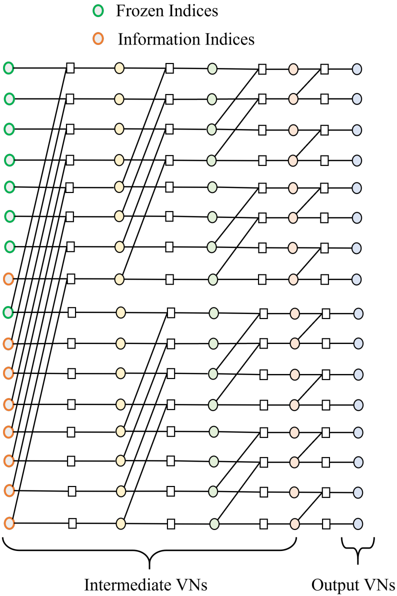

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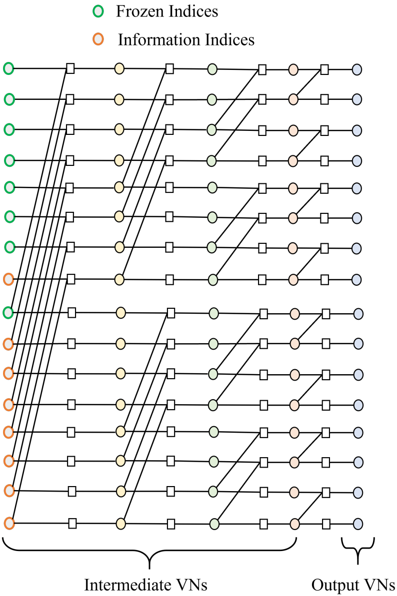

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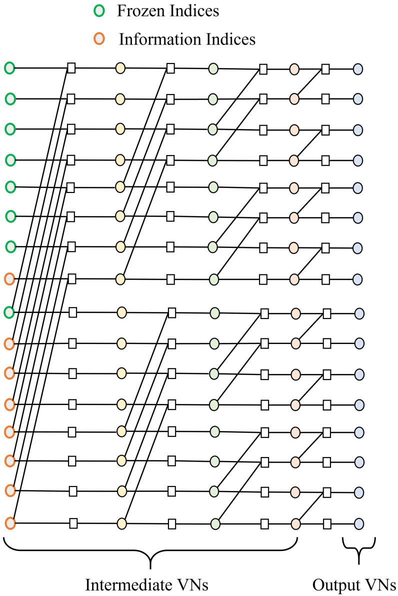

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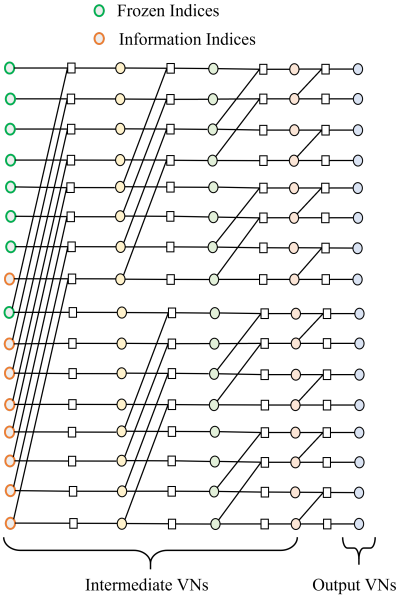

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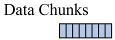
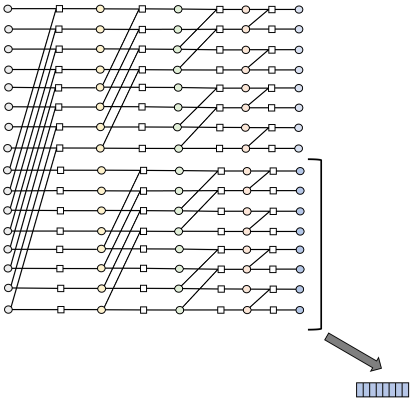
PCMT
- store the hashes of the intermediate VNs
- use these hashes to build small IC-proofs for the degree 2 and degree 3 CNs

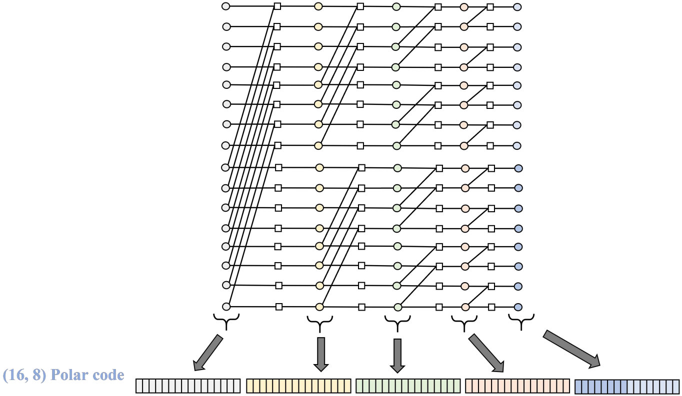
PCMT Construction



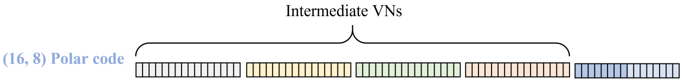
PCMT Construction



PCMT Construction



PCMT Construction



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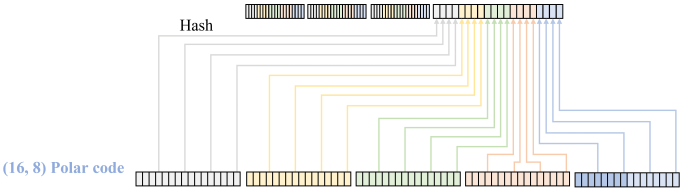

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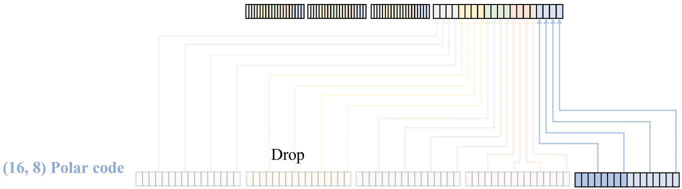

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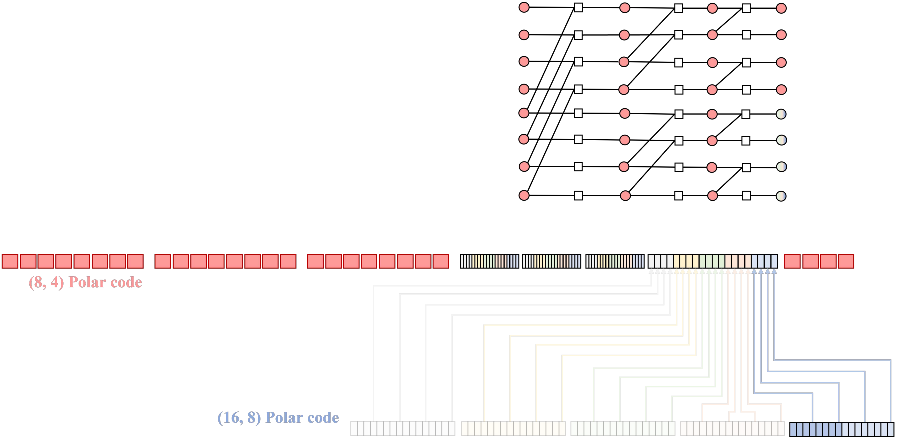

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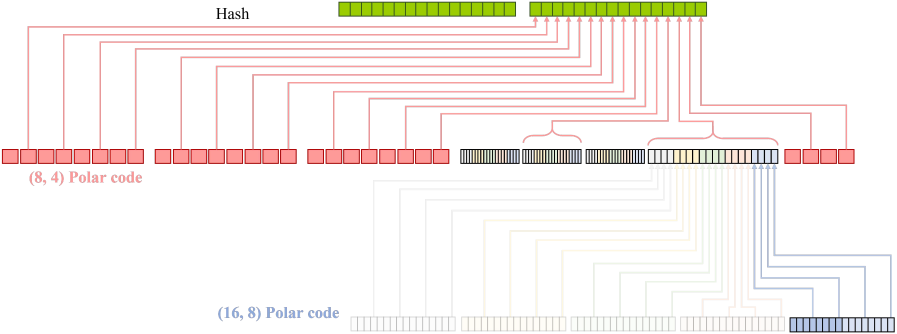

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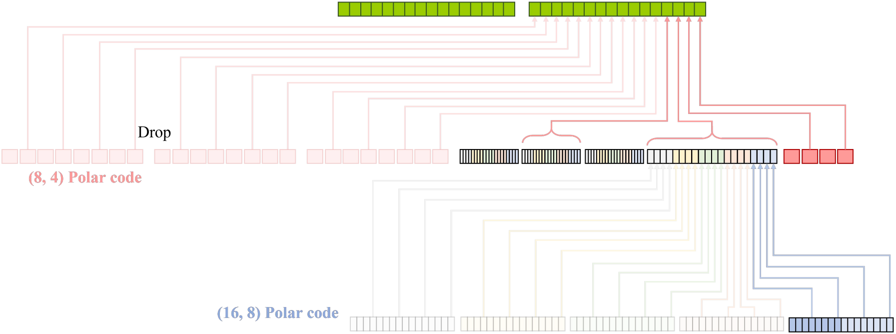

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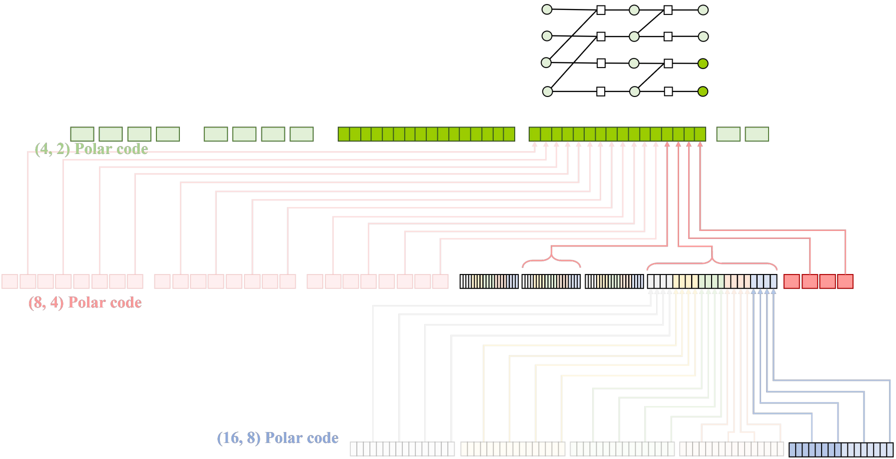

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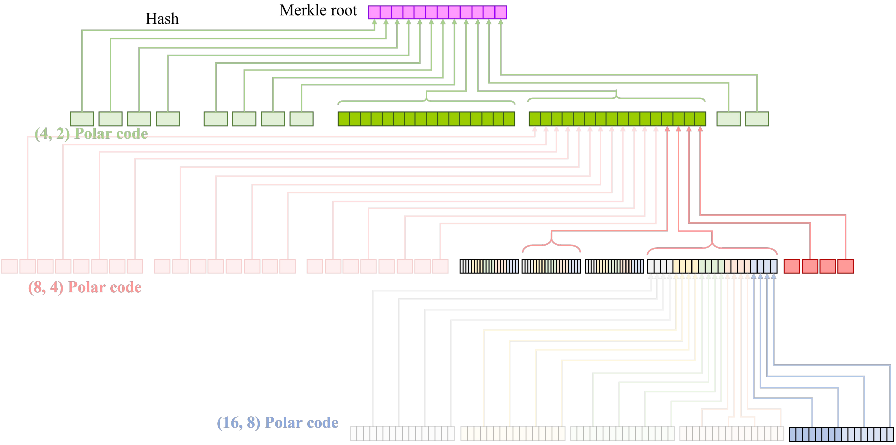

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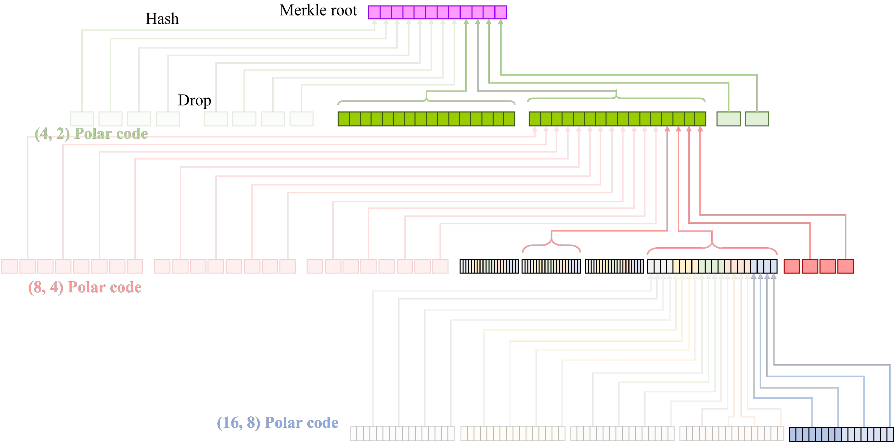

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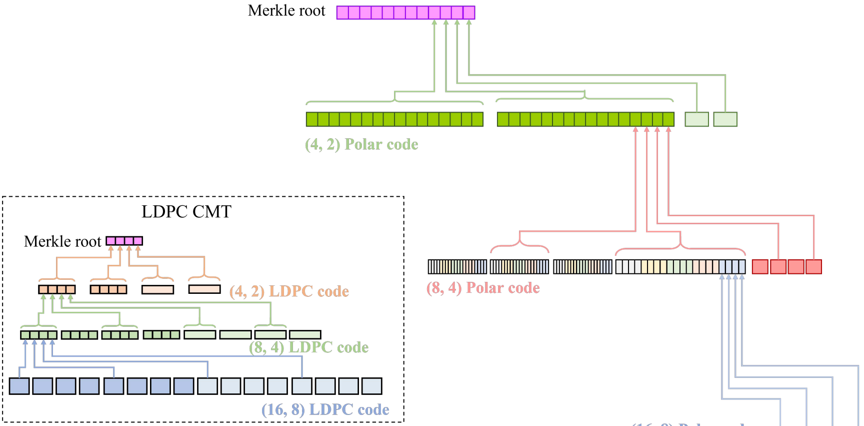

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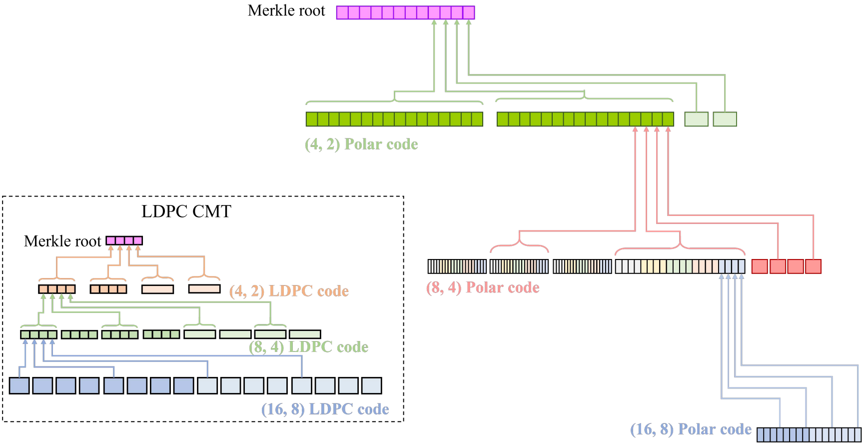
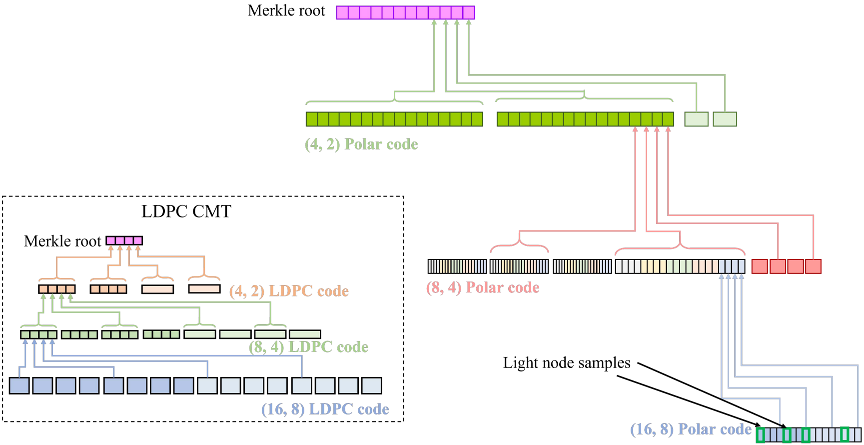


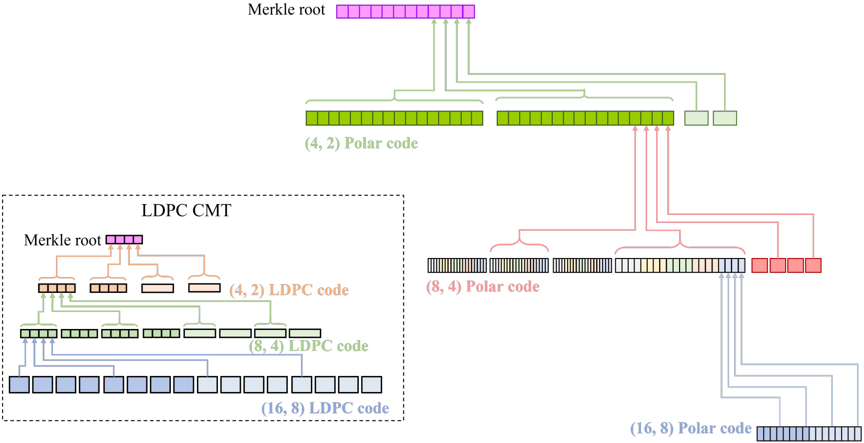
- Dropped VNs can be decoded back using a peeling decoder

PCMT Construction



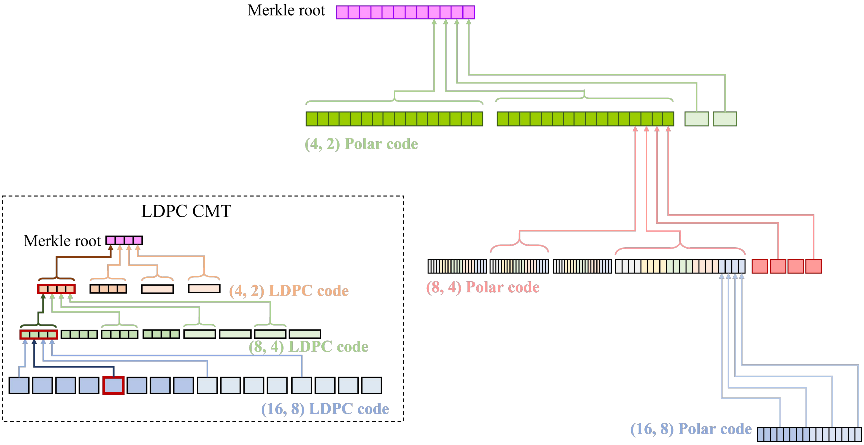
- Dropped VNs can be decoded back using a peeling decoder
- Light nodes sample the non-dropped VNs

PCMT: Merkle Proofs



- Both dropped and non-dropped VNs have merkle proofs

PCMT: Merkle Proofs



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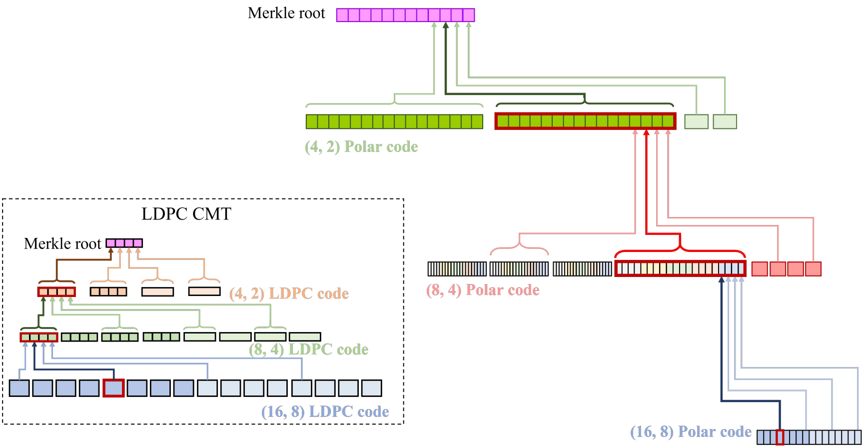
PCMT: Merkle Proofs



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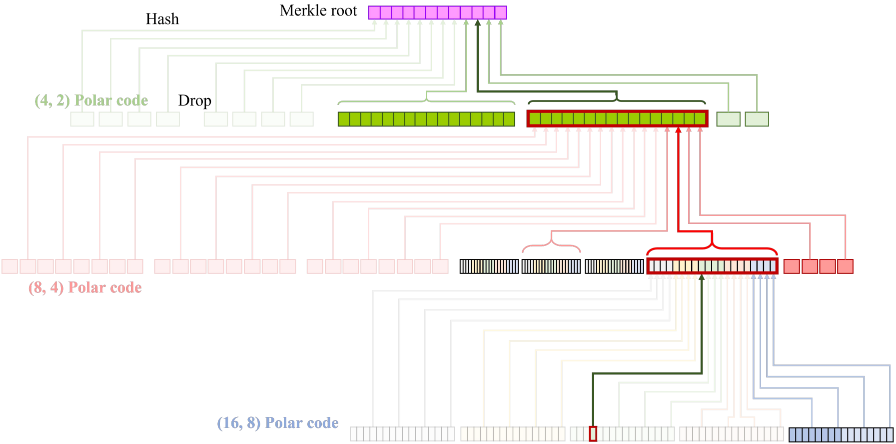
PCMT: Merkle Proofs



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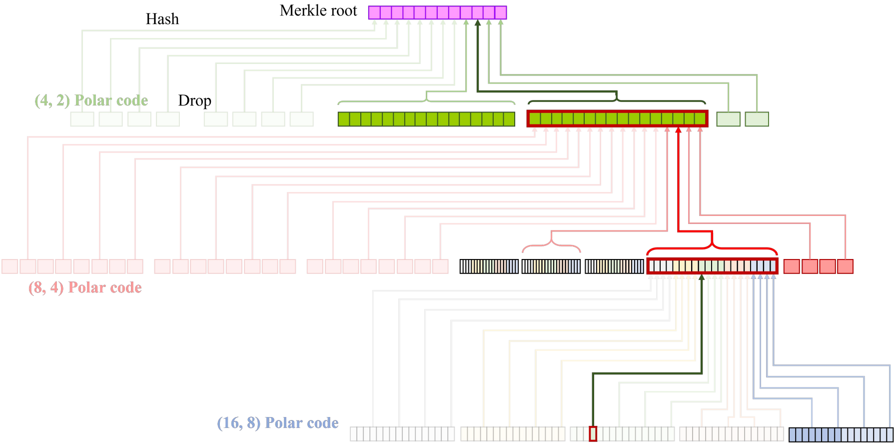
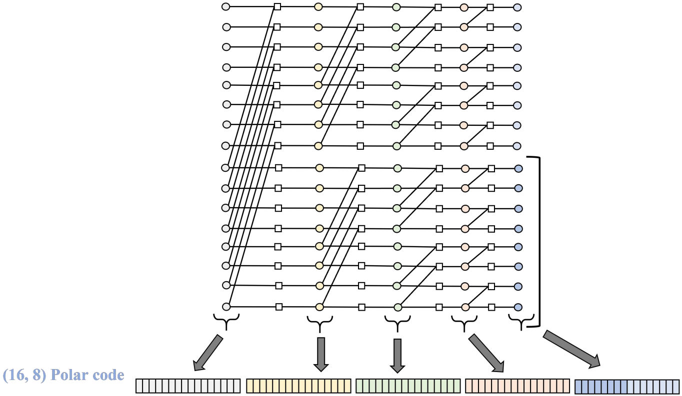
PCMT: Merkle Proofs



- Both dropped and non-dropped VNs have merkle proofs
- Used for integrity checks and in IC-proofs similar to LDPC CMT

Frozen Index Selection for PCMT



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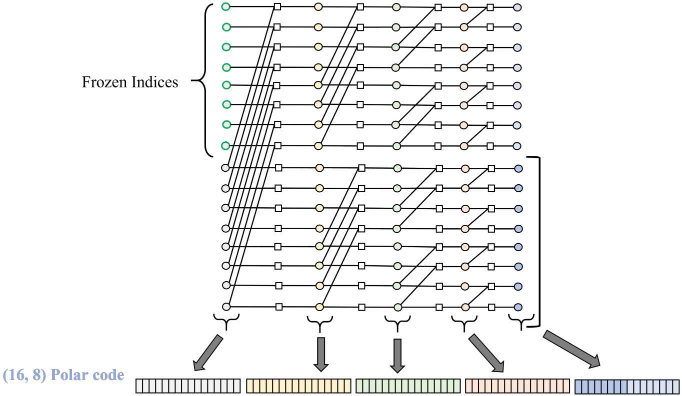
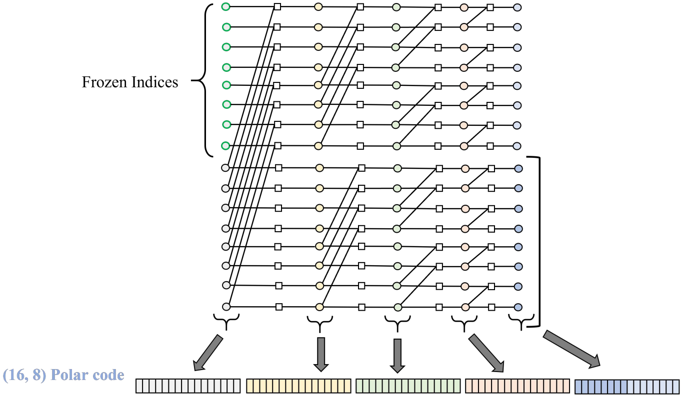
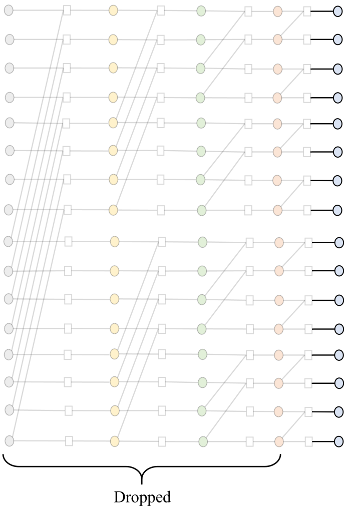


Frozen Index Selection for PCMT



- Not the best choice for frozen indices

Frozen Index Selection for PCMT



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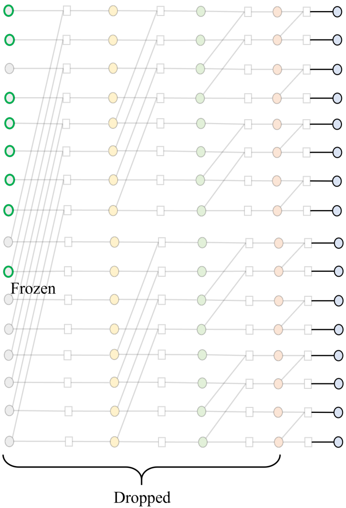
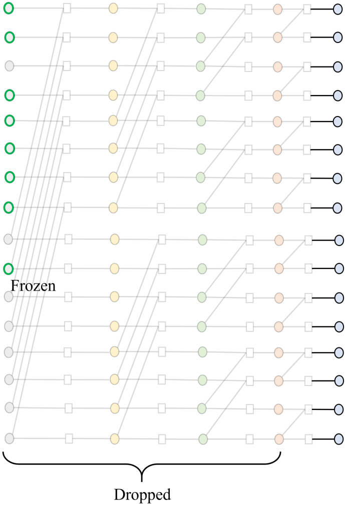


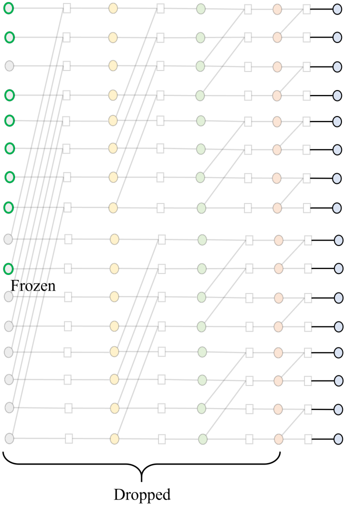
Frozen Index Selection for PCMT



Adversary:

- Cannot hide frozen VNs

Frozen Index Selection for PCMT



Adversary:

- Cannot hide frozen VNs
- Must hide non-dropped VNs such that a stopping set becomes unavailable

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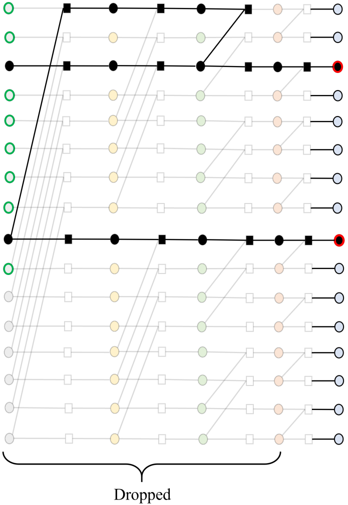

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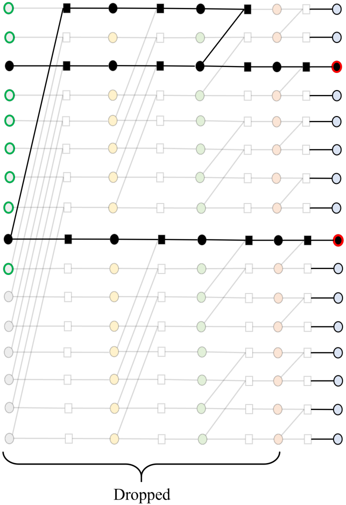
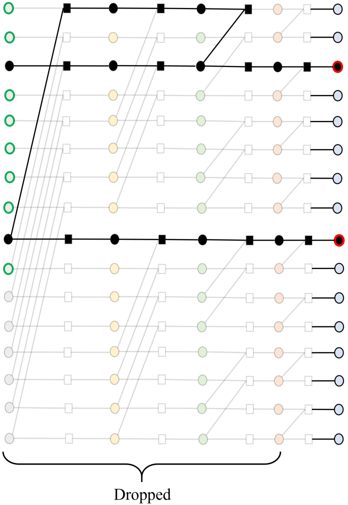


Adversary:

- Cannot hide frozen VNs
- Must hide non-dropped VNs such that a stopping set becomes unavailable
- Hide the leaf set of a stopping set with no frozen VNs

Frozen Index Selection for PCMT



Adversary:

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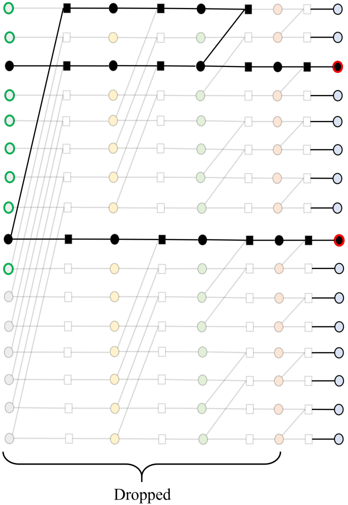
Undecodable threshold $\alpha_{\min}$

Frozen Index Selection for PCMT

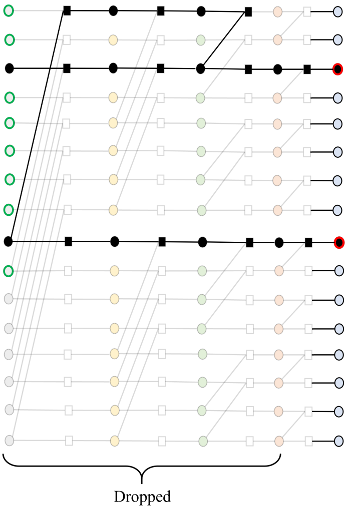


Adversary:

- Cannot hide frozen VNs
- Must hide non-dropped VNs such that a stopping set becomes unavailable
- Hide the leaf set of a stopping set with no frozen VNs

Undecodable threshold $\alpha_{\min}$ $=$ smallest leaf set size of all stopping sets with no frozen VNs

Frozen Index Selection for PCMT



Adversary:

- Cannot hide frozen VNs
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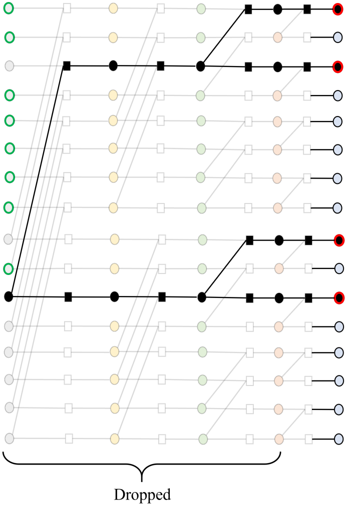
Undecodable threshold $\alpha_{\min}$
= smallest leaf set size of all stopping sets with no frozen VNs
= smallest leaf set size of all stopping trees with no frozen VNs [Eslami '13]

Frozen Index Selection for PCMT



Adversary:

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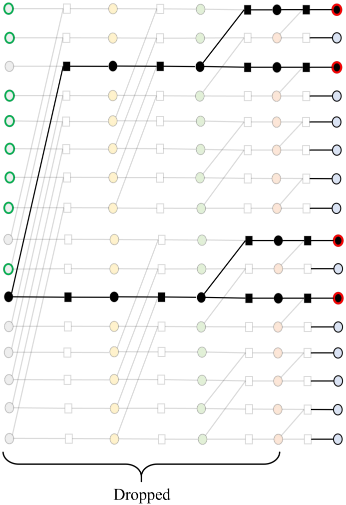
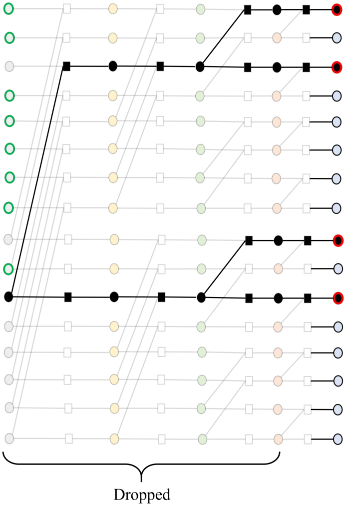
Frozen Index Selection for PCMT



Stopping Trees:

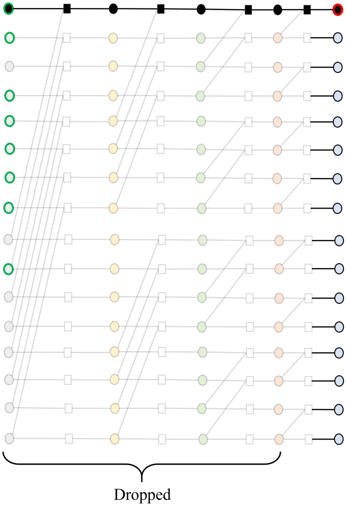
Frozen Index Selection for PCMT



Stopping Trees:

- Every VN in the leftmost column is associated with a unique stopping tree

Frozen Index Selection for PCMT



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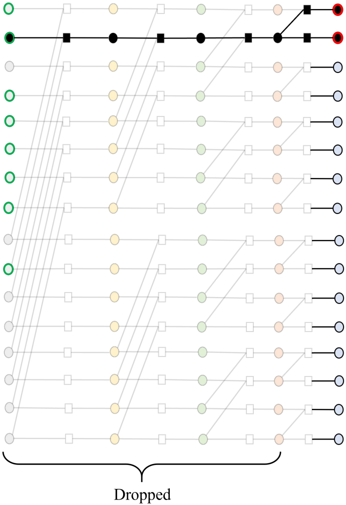

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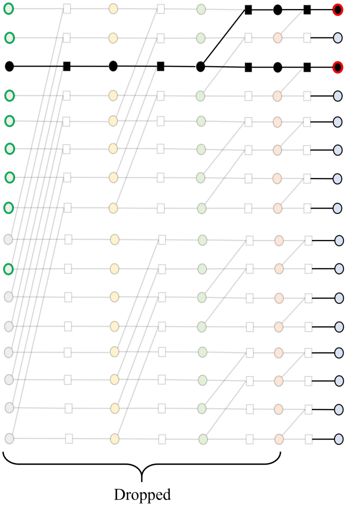

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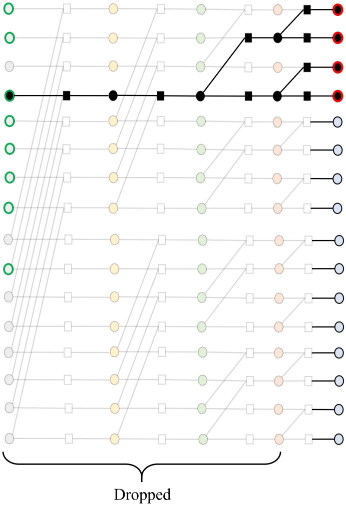

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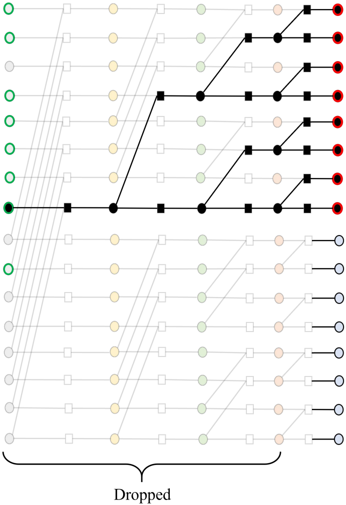

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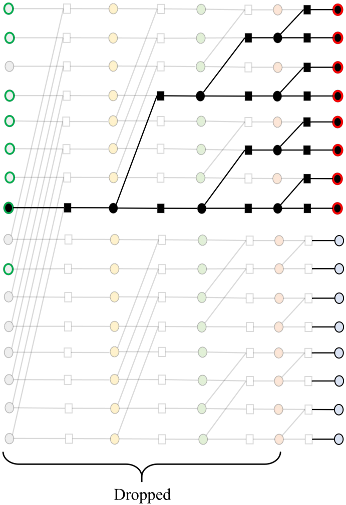

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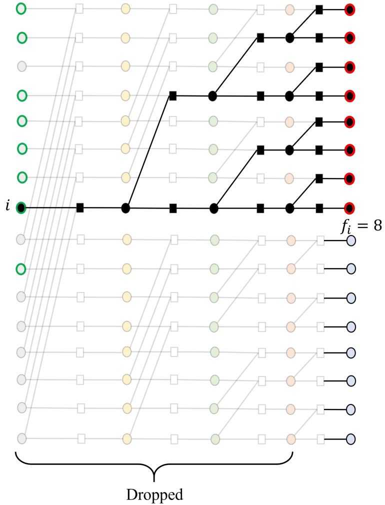

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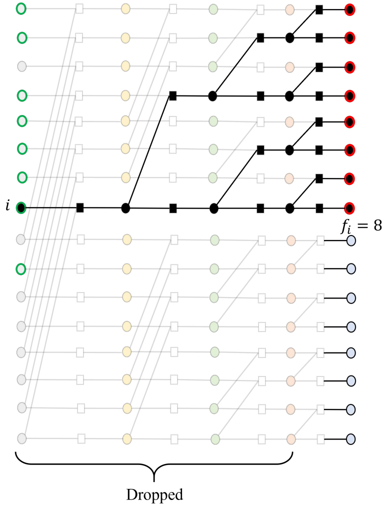

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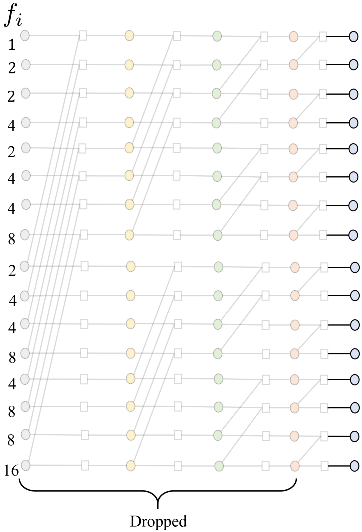

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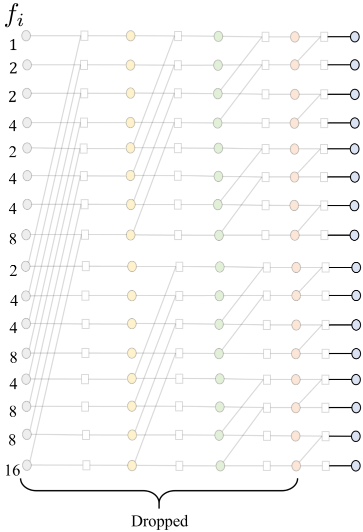

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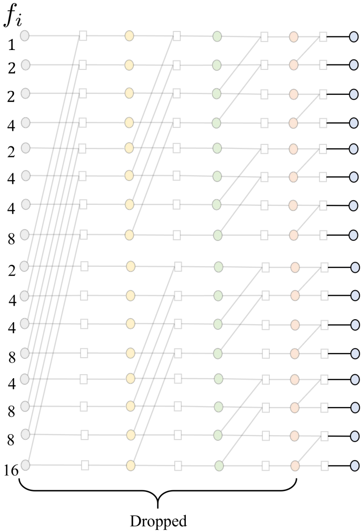
Naive selection method

Frozen Index Selection for PCMT



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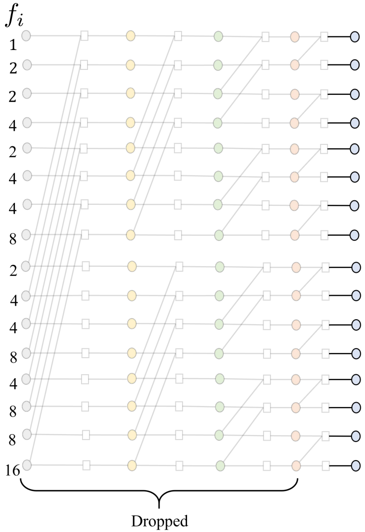
Naive selection method
- For (N, k) polar code, select the indices with $N - k$ smallest leaf set sizes as frozen set.

Frozen Index Selection for PCMT



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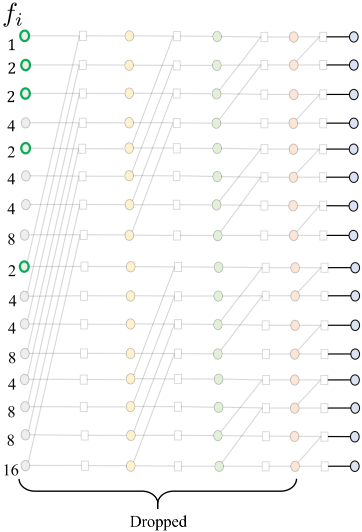
E.g. $(16, 8)$ polar code:

Frozen Index Selection for PCMT



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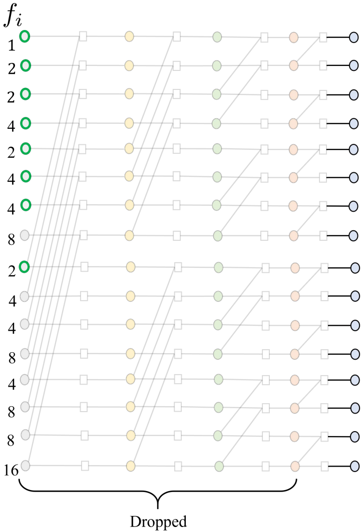

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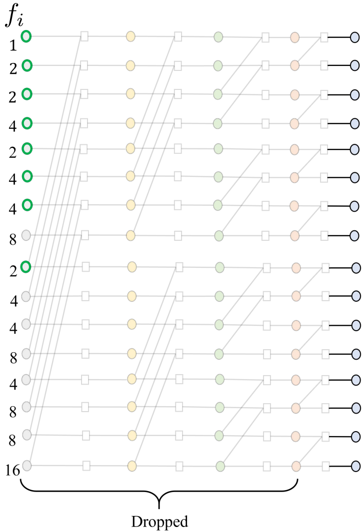

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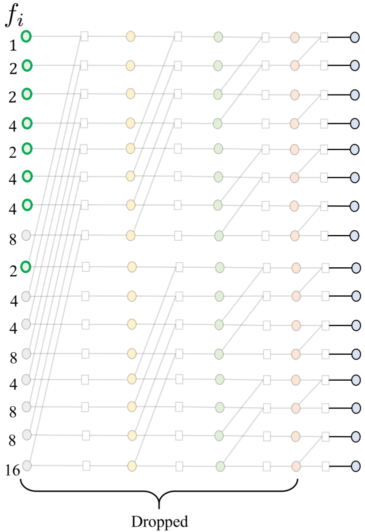
E.g. $(16, 8)$ polar code: $\alpha_{\min} = 4$.

Frozen Index Selection for PCMT



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Naive selection method
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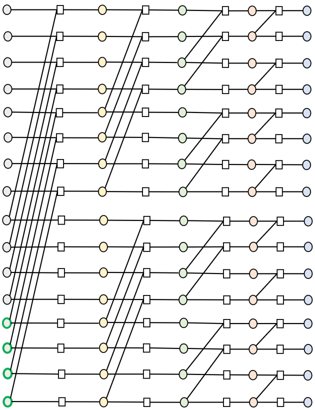
E.g. $(16, 8)$ polar code: $\alpha_{\min} = 4$.

Can we do better ?

Sampling Efficient Freezing (SEF) Algorithm

Lemma

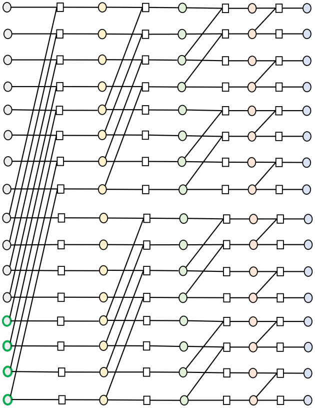
If we freeze last μ indices from the bottom of the encoding graph,



Sampling Efficient Freezing (SEF) Algorithm

Lemma

If we freeze last μ indices from the bottom of the encoding graph, then a stopping set with no frozen VNs cannot have a VN from the last μ rows of the encoding graph.



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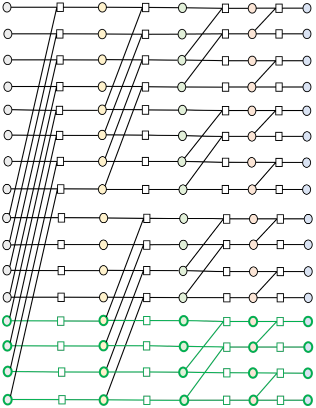

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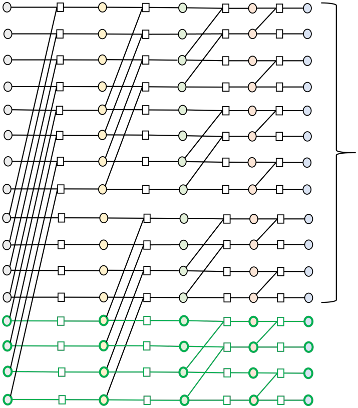
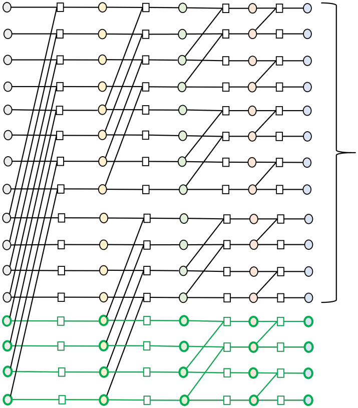


- Light nodes do not need to sample VNs from the last μ rows

Sampling Efficient Freezing (SEF) Algorithm

Lemma

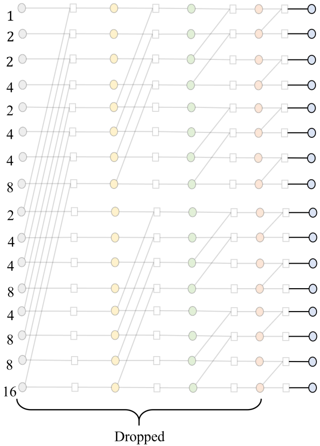
If we freeze last μ indices from the bottom of the encoding graph, then a stopping set with no frozen VNs cannot have a VN from the last μ rows of the encoding graph.



- Light nodes do not need to sample VNs from the last μ rows
  -Improves the effective undecodable threshold

Sampling Efficient Freezing (SEF) Algorithm

Design of (16, 8) Polar code:



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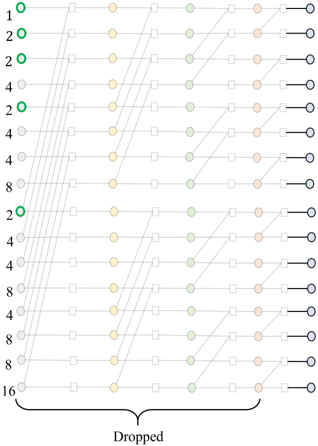

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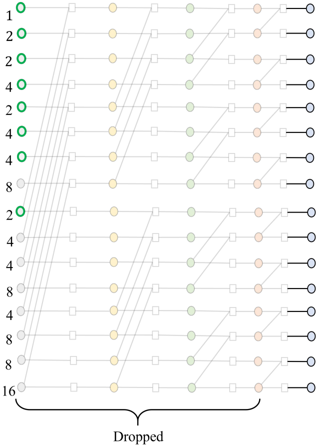

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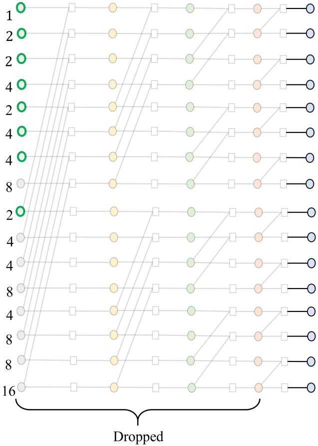

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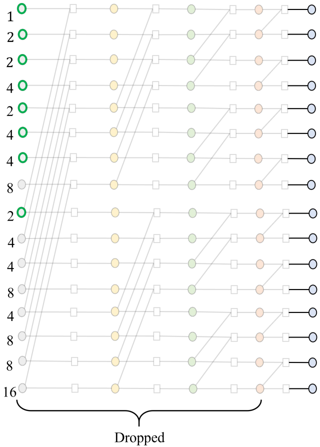

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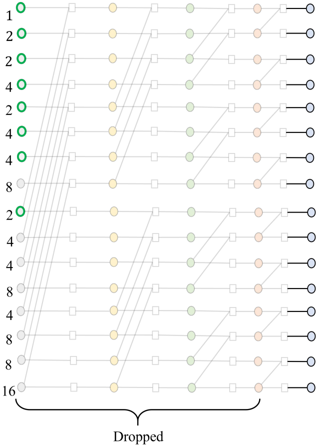

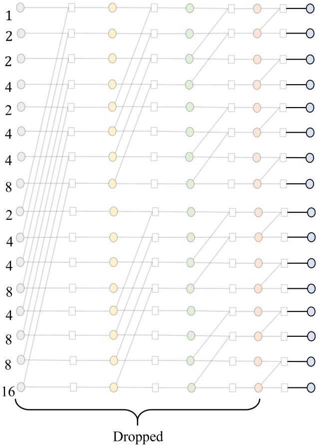

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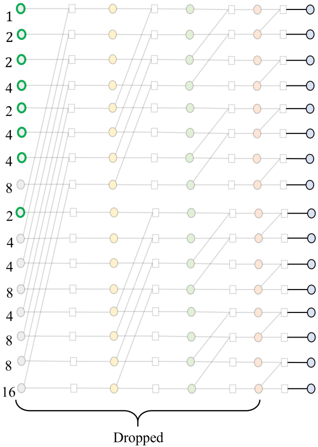

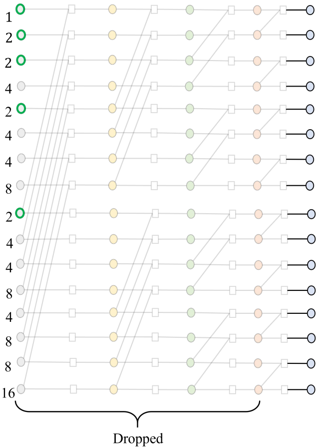

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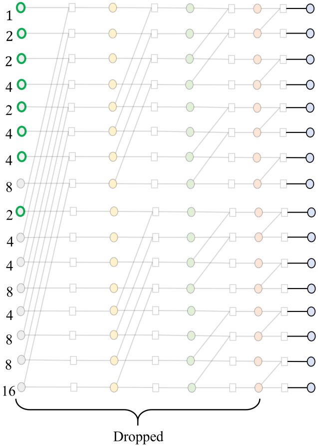

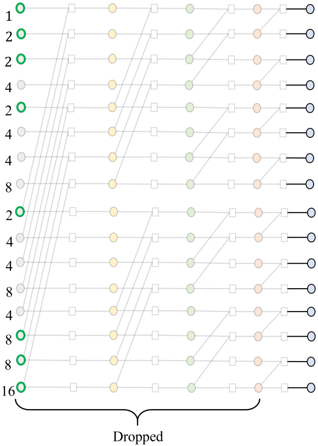

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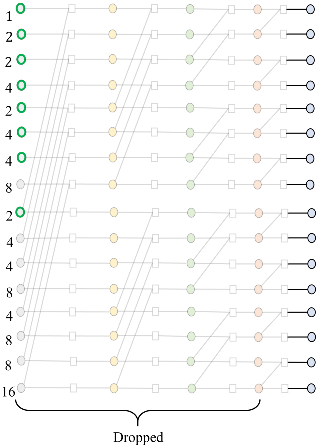

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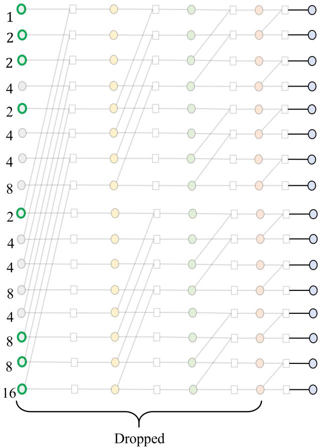

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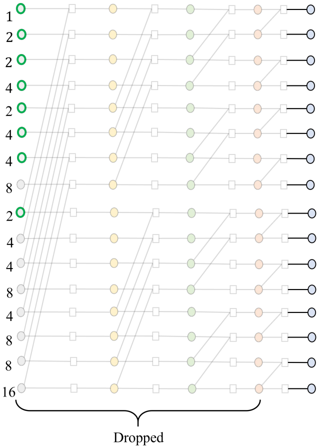

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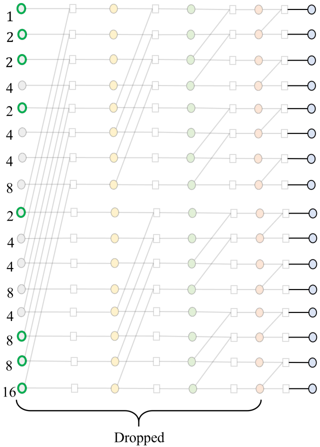

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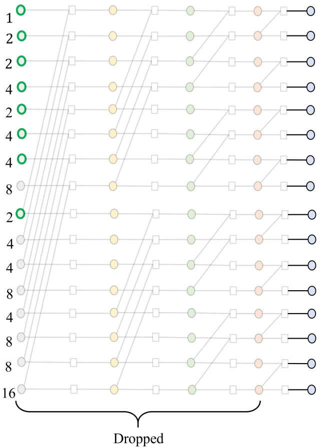

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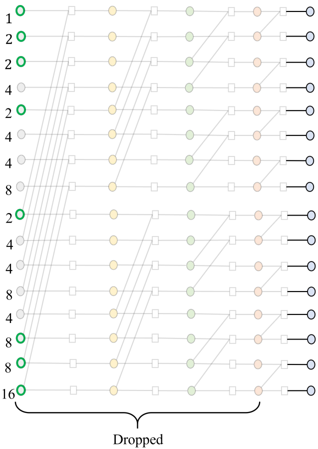

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- $\alpha_{\min}^{\text{effective}} = \frac{4*16}{13} = 4.923$

Simulation Results: IC-proof size

Parameters: Rate $R = 0.5$, Code length N, Data chunk size c,
Block size $b = cRN$, Hash size $= 32B$

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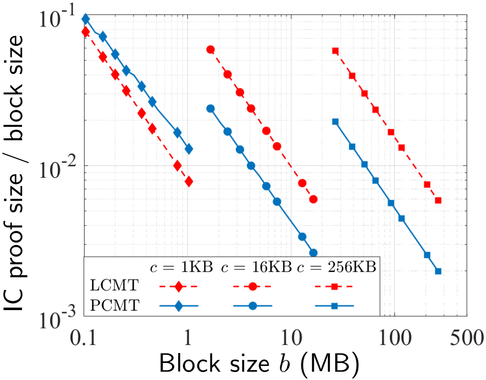
LCMT: LDPC CMT



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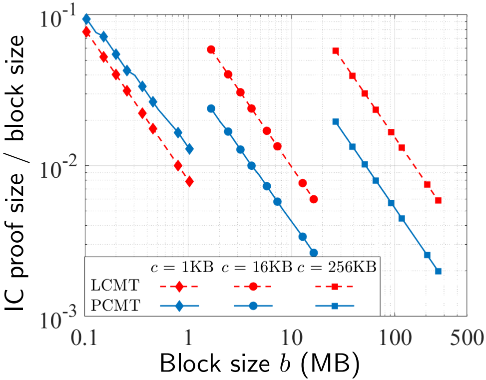

- For large block sizes, IC-proof size of PCMT is lower than LCMT

Simulation Results: Probability of failure

Code length N, Data chunk size $c = 256KB$, Hash size $32B$, Block size $b = cRN$, number of samples s such that total sample download is $\frac{b}{5}$

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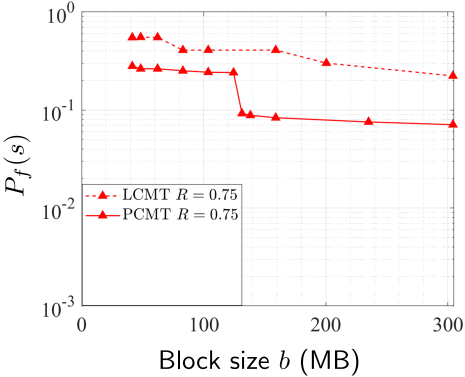

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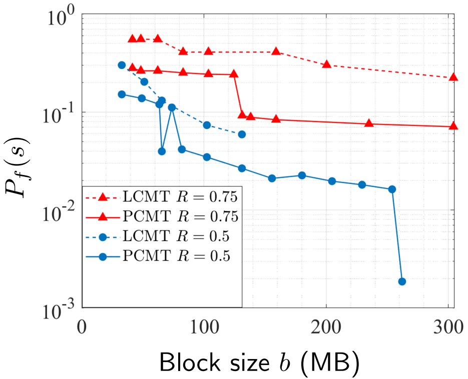

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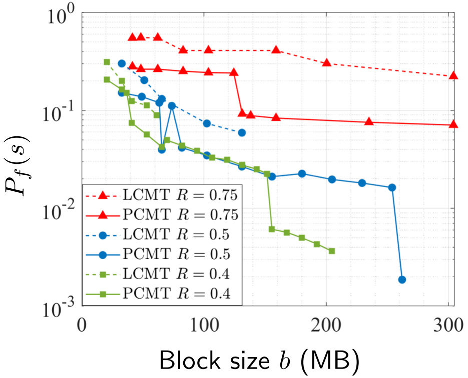

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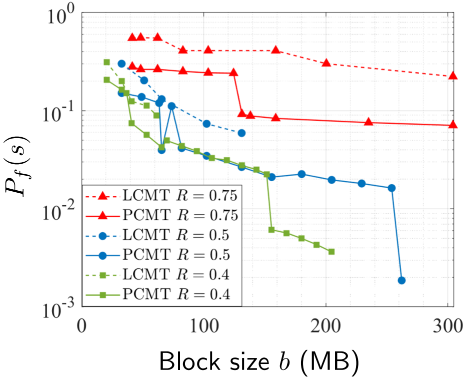

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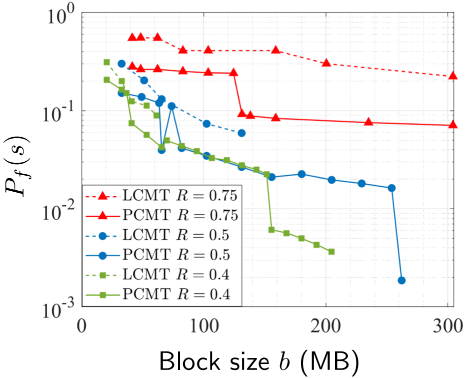

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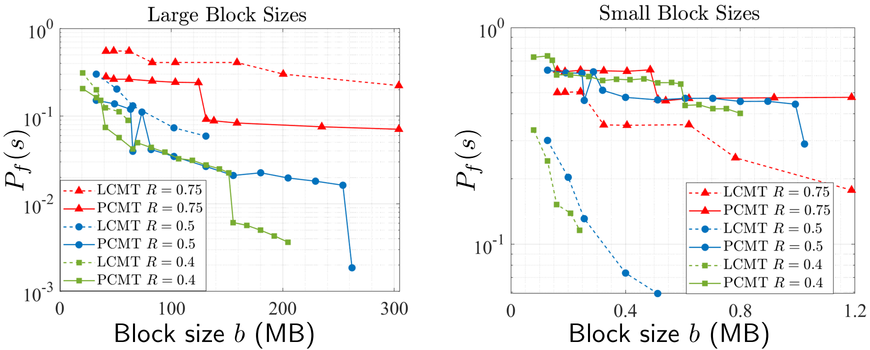

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Future Work:

- Improve the PCMT construction to make it not store hashes of all VNs of the encoding graph
- Extend the PCMT construction to other encoding trellises

References

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Simulation Results

$\mathcal{T}_1$: small block size
$\mathcal{T}_2$: large block size

	2D-RS [Al-Bassam '18] [Santini '22]		LCMT		PCMT	
	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_1$	$\mathcal{T}_2$	$\mathcal{T}_1$	$\mathcal{T}_2$
Root size (KB)	2.05	5.82	0.26	0.51	1.02	2.56
IC proof size (MB)	5.80	16.40	1.54	1.54	0.53	0.54
Undecodable threshold $\alpha_{\min}$	Analytical expression		NP-hard		Analytical expression	
Decoding complexity	$O(N^{1.5})$		$O(N)$		$O(N\lceil\log N\rceil)$	

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- PCMT offers a new trade-off in the metrics of importance compared to LCMT and 2D-RS codes that were used in prior literature