Concentrated Stopping Set Design for Coded Merkle Tree: Improving Security Against Data Availability Attacks in Blockchain Systems

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Only store block headers



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- Rely on honest full nodes for fraud notification



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Adversary creates an invalid block



Full nodes: cannot send fraud proof



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- Full nodes: cannot send fraud proof
- Light nodes: accept the invalid header



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Our work: Design of specialized LDPC codes with a coupled sampling strategy to achieve a significantly lower probability of failure.

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Concentrating cycles ⇒
 Concentrating stopping sets

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How to concentrate stopping sets?

- Concentrating cycles ⇒
 Concentrating stopping sets
- We concentrate cycles by modifying the Progressive Edge Growth (PEG) algorithm

EC (Entropy Constrained)-PEG Algorithm For each VN v_j

Expand Tanner Graph in a BFS fashion

- If \exists CNs not connected to v_j
 - select a CN with min degree not connected to v_j

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Entropy to Concentrate Cycles: EC-PEG Algorithm For distribution $p = (p_1, p_2, ..., p_n)$, Entropy $\mathcal{H}(p) = \sum_{i=1}^n p_i \log \frac{1}{p_i}$

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Sampling Strategy

- Greedy Sampling: greedily sample VNs that are part of a large number of cycles
- Random Sampling (with replacement): sample each variable node with equal probability

Probability of failure for a stopping set of size $\boldsymbol{\mu}$



RS: Random Sampling





Probability of failure for a stopping set of size μ





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Concentrated LDPC codes with Greedy sampling improve the probability of failure

References

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