Concentrated Stopping Set Design for Coded Merkle Tree: Improving Security Against Data Availability Attacks in Blockchain Systems

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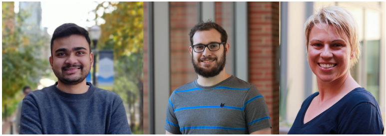
ITW 2020

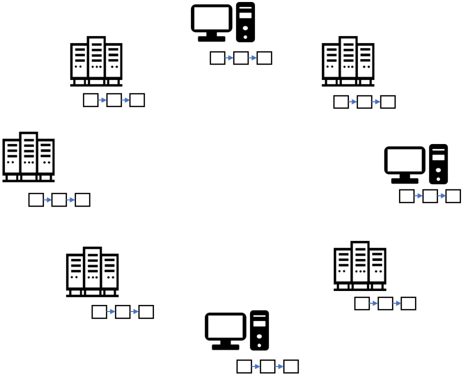


UCLA Samueli School of Engineering

Blockchain

- Distributed Ledger
- Decentralized trust platforms
- Application:
  - Finance and currency
  - Healthcare services
  - Supply chain management
  - Industrial IoT
  - e-voting

Central Problem: Prohibitive Storage Overhead



- Ledger maintained by a network of nodes

[1] As of 3/12/2021, https://bitinfocharts.com/

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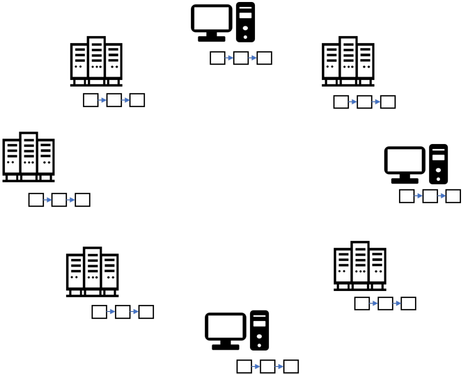

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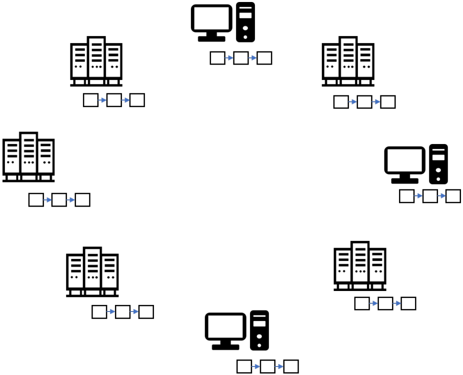

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Significant storage overhead

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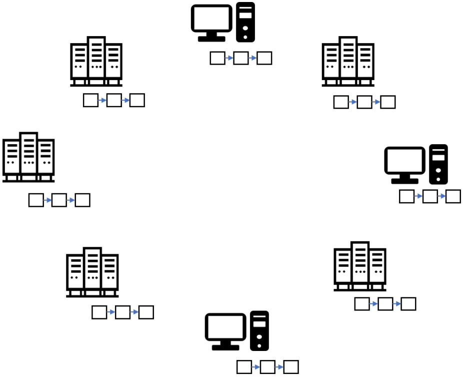

Significant storage overhead

- Ledger maintained by a network of nodes
- Each node maintains a local copy of the ledger

- Bitcoin ledger size $\sim$ 350GB[1]
- Ethereum ledger size $\sim$ 600GB[1]

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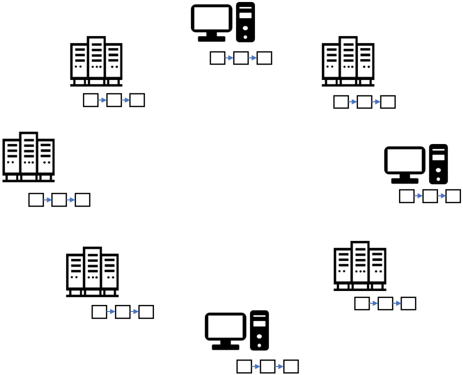

Significant storage overhead

- Ledger maintained by a network of nodes
- Each node maintains a local copy of the ledger
- Prohibitive for resource limited nodes

- Bitcoin ledger size ∼ 350GB[1]
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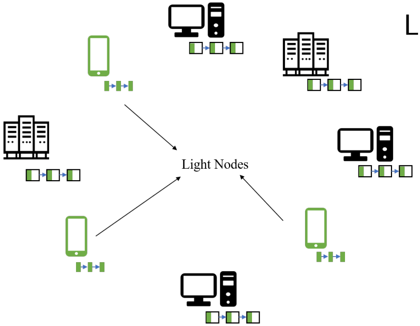
Allowing Light Nodes



Light Nodes:

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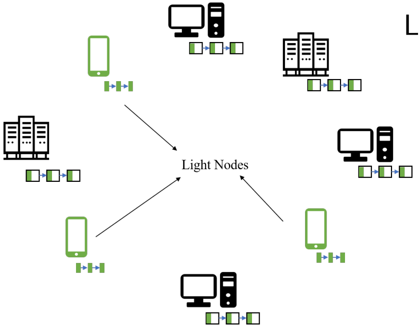

Light Nodes:

- Only store block headers
  (total size $\sim$ 1GB for Ethereum)

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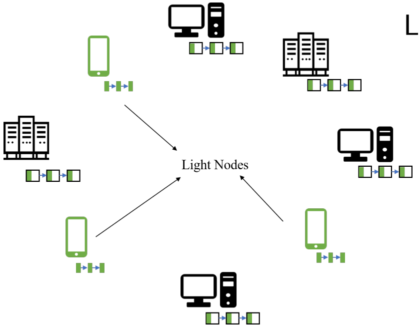

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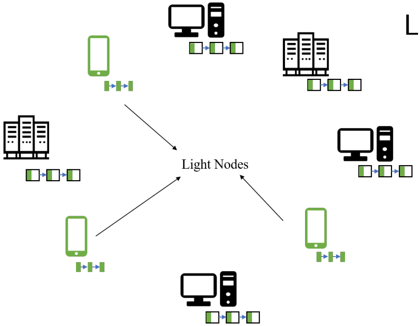

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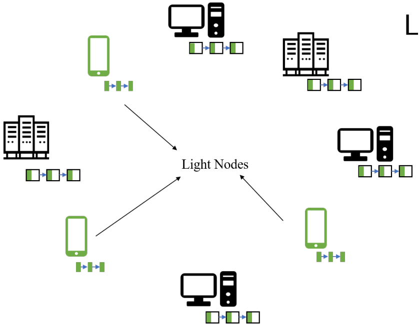


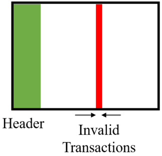
Light Nodes:

- Only store block headers (total size $\sim$ 1GB for Ethereum)
- Can verify transaction inclusion in a block
- Cannot verify transaction correctness $\rightarrow$ Rely on honest Full nodes for fraud notification

Data Availability(DA) Attack

Systems with light nodes and a dishonest majority of full nodes are vulnerable to DA attacks [Al-Bassam '18], [Yu '19]

Adversary creates an invalid block



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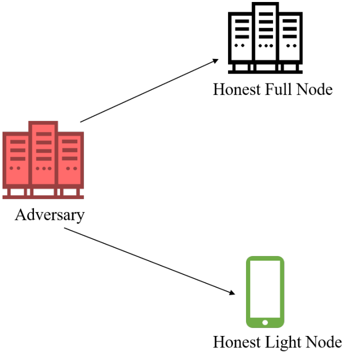

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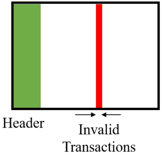

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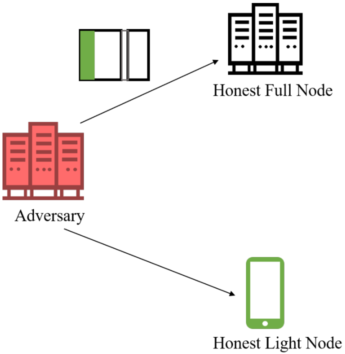

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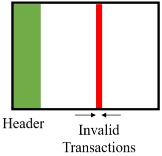

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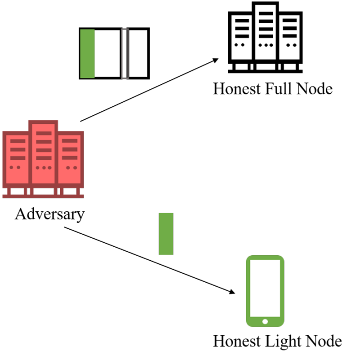

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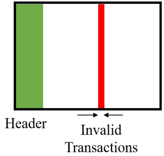

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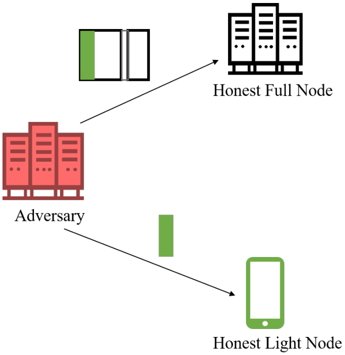

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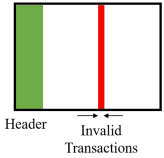

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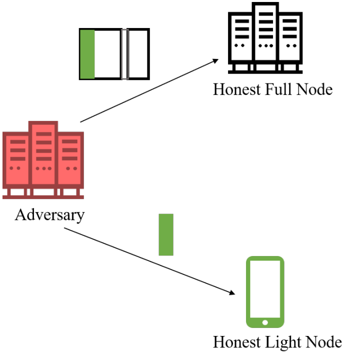

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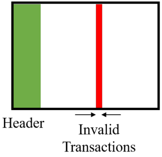

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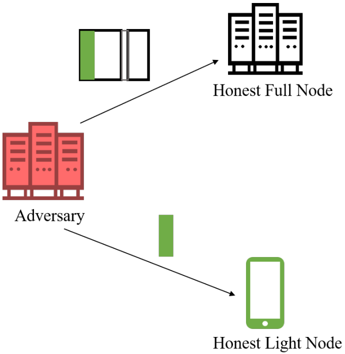

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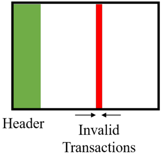

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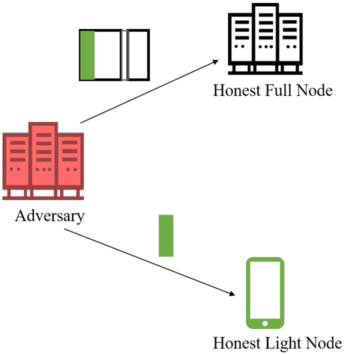

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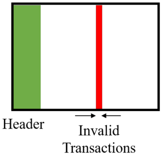

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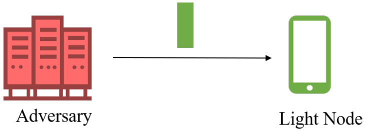
- Light Nodes: No fraud proof $\rightarrow$ accept the header.

Ensuring Data Availability



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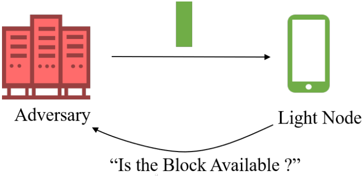

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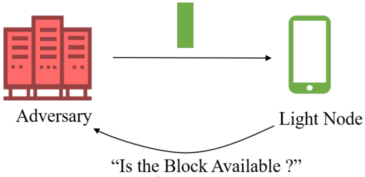

- Anonymously request/sample few random chunks of the block

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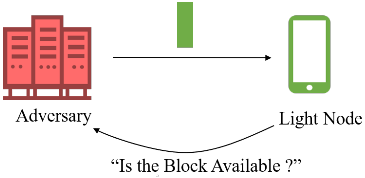

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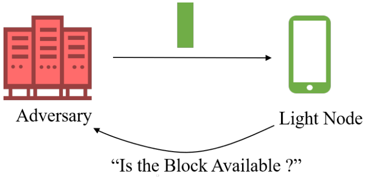

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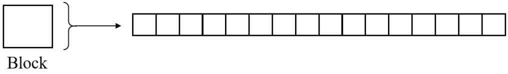

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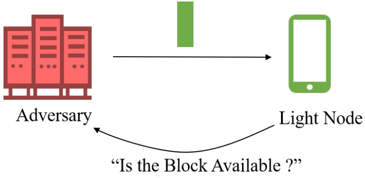

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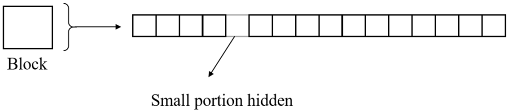

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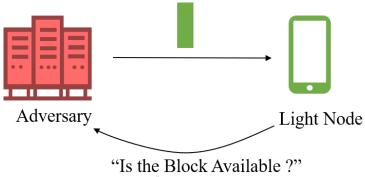

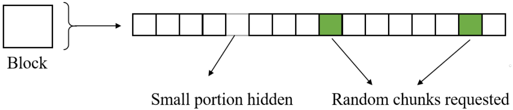

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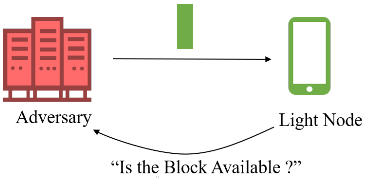

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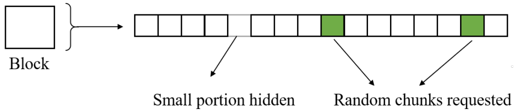
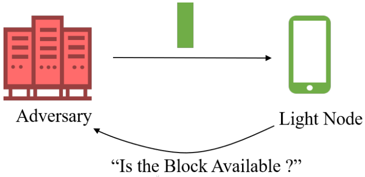


Probability of failure using 2 random samples:

Ensuring Data Availability



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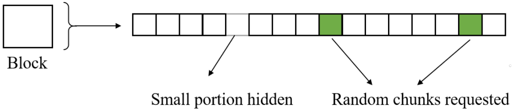
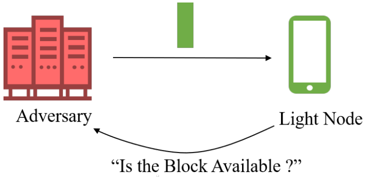


Probability of failure using 2 random samples:

$$\left(1-\frac{1}{16}\right)\left(1-\frac{1}{15}\right)=0.87$$

Ensuring Data Availability



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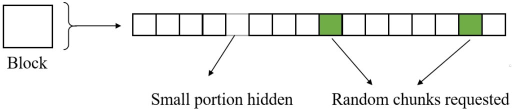
No coding:



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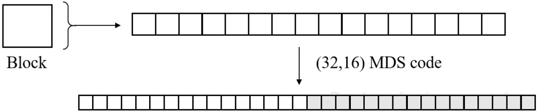

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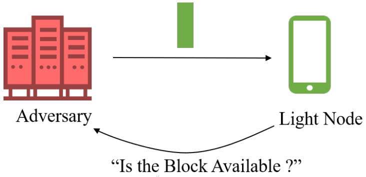

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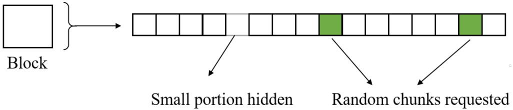

Probability of failure using 2 random samples:

$$\left(1 - \frac{1}{16}\right)\left(1 - \frac{1}{15}\right) = \color{red}{0.87}$$

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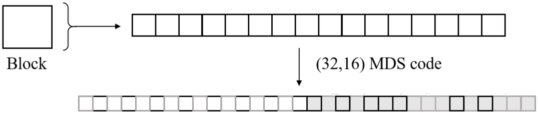

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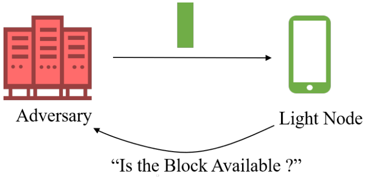

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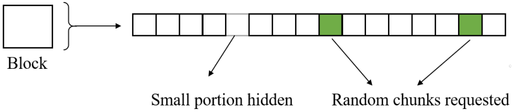

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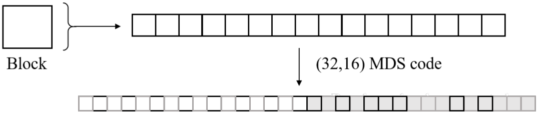

Probability of failure using 2 random samples:

$$\left(1 - \frac{17}{32}\right)\left(1 - \frac{17}{31}\right) = 0.21$$

Choice of Code Matters

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- Incorrect coding attack:

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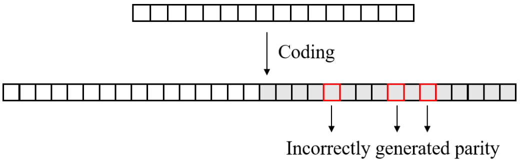

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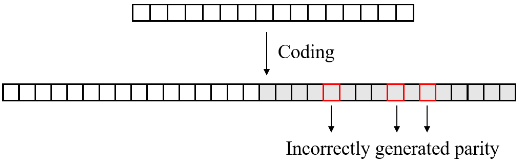

- Incorrect coding attack:
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  - Honest Full nodes can detect and send incorrect coding proof
  - Incorrect coding proof size: $\mathcal{O}$(sparsity of parity check equations)

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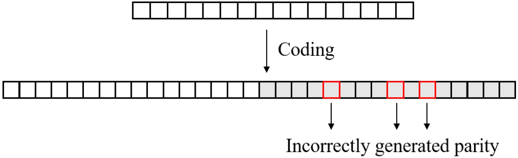

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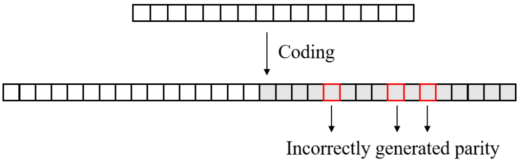

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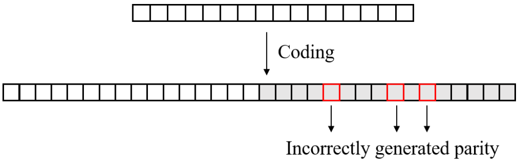

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- Undecodable ratio α

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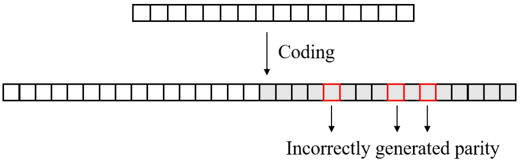

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- Undecodable ratio α
  - Probability of Light node failure using s random samples $= (1-\alpha)^s$

LDPC Codes: A Strong Contender

LPDC codes:

- Characterized by a sparse parity check matrix

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circles: variable nodes (VNs)
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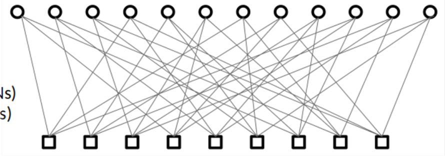

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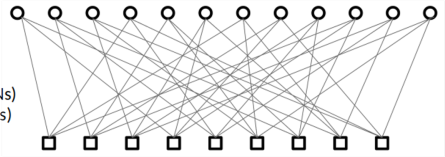

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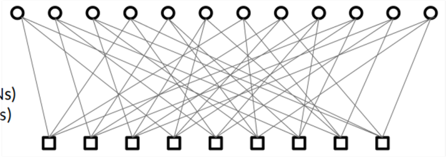

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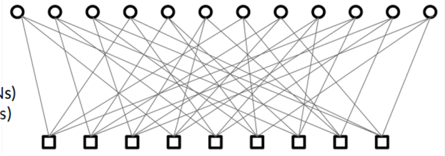

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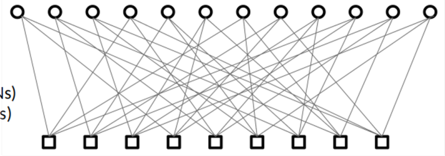

LDPC codes have been shown to be suitable for this application [Yu' 19]

- Small incorrect coding proof size due to small check node degree
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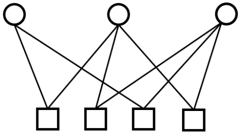
- What about the undecodable ratio?

Challenge with LDPC Codes: Small Stopping Sets

- Substructure in the Tanner Graph



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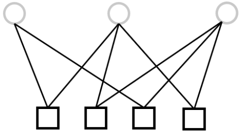
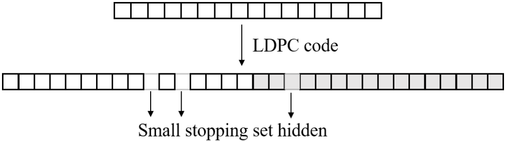
- If hidden, prevents peeling decoder from decoding the block $\rightarrow$ No fraud proof





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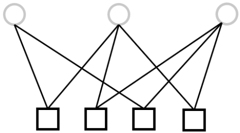

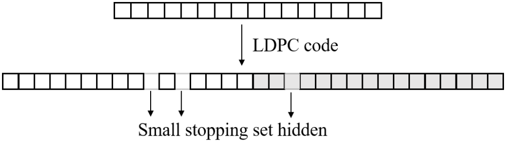

Probability of failure using 2 random samples:

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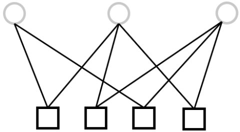

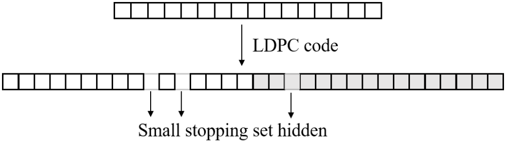

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> Our work: Design of specialized LDPC codes with a coupled sampling strategy to achieve a significantly lower probability of failure.

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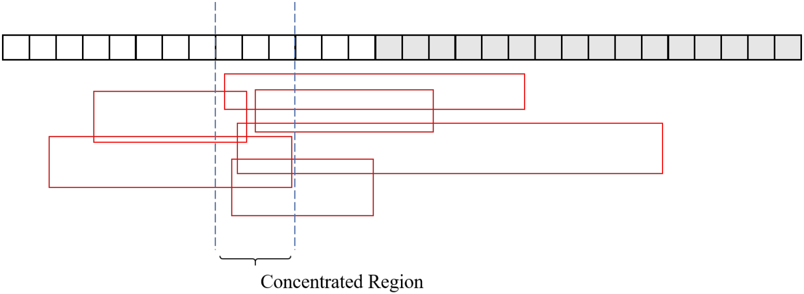
- Selecting a set $\mathcal{L}$ of VNs which touches large no. of SSs $\rightarrow$ Prob. of failure $\downarrow$

Concentrated Stopping Set Design



Code Design Idea:

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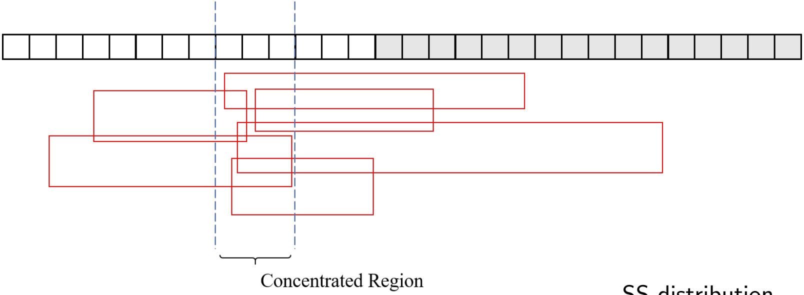

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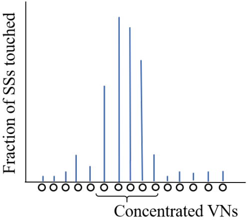

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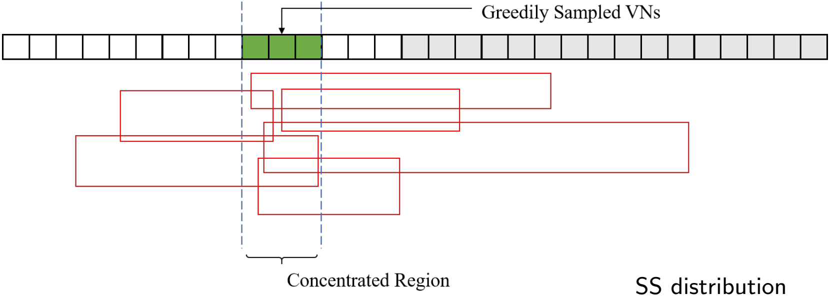
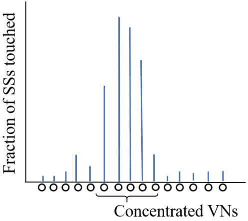


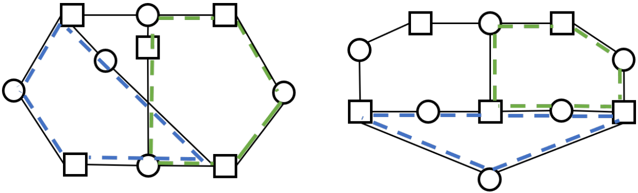
Code Design Idea:

- Concentrate stopping sets to a small section of VNs
- Greedily Sample this small section of VNs

SS distribution



How to Concentrate Stopping Sets?



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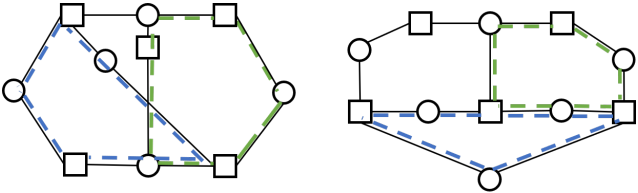

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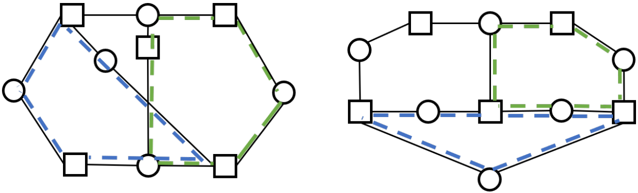

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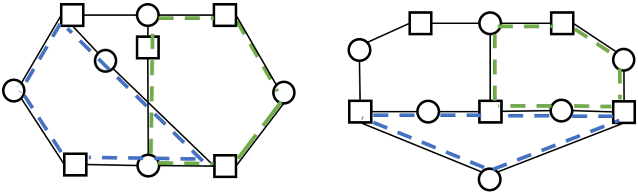

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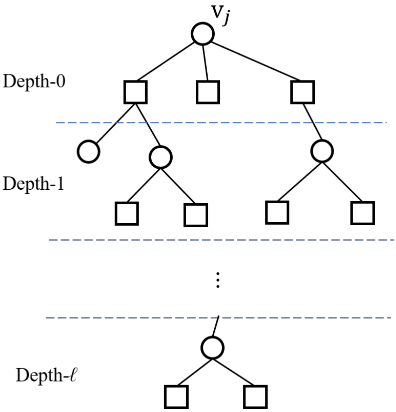
- How to design codes with concentrated cycles?
  We do so by modifying the well-known Progressive Edge Growth (PEG) algorithm

PEG Algorithm

- Constructs a Tanner Graph in an edge by edge manner [Xiao '05]

PEG Algorithm



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For each VN v_j
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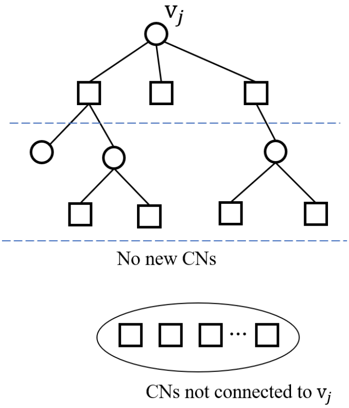

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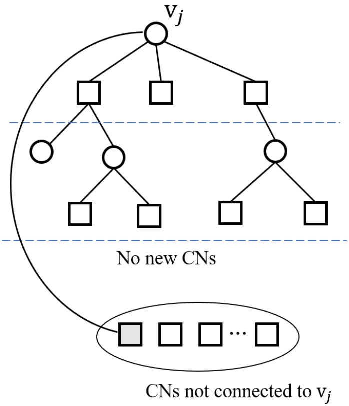

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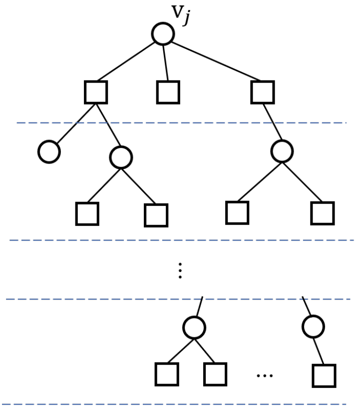

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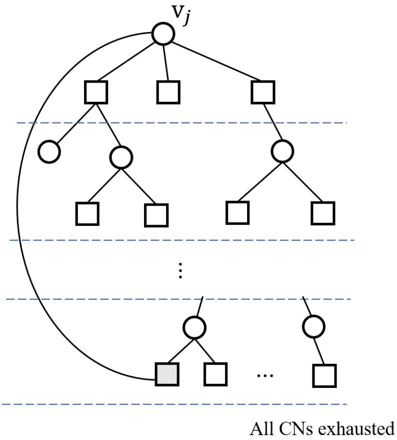

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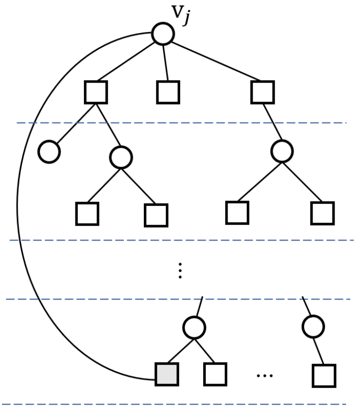
PEG Algorithm



All CNs exhausted

- Constructs a Tanner Graph in an edge by edge manner [Xiao '05]

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New cycles created

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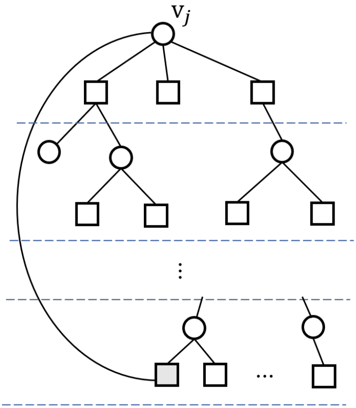

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New cycles created

We modify the CN selection criteria in green to concentrate cycles

Using Entropy to Concentrate Cycles

For distribution $p = (p_1, p_2, \ldots, p_n)$, Entropy $\mathcal{H}(p) = \sum_{i=1}^{n} p_i \log \frac{1}{p_i}$

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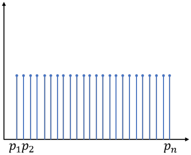

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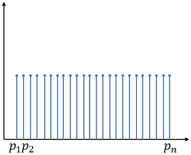

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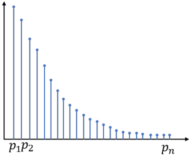

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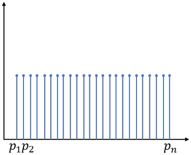

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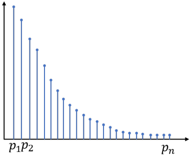

Low Entropy

We want the cycle distributions to be concentrated

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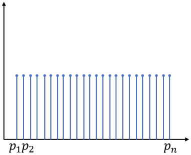

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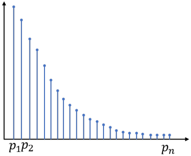

Low Entropy

We want the cycle distributions to be concentrated
$\rightarrow$ Select CNs such that the entropy of the cycle distribution is minimized

Using Entropy to Concentrate Cycles

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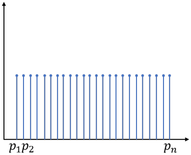

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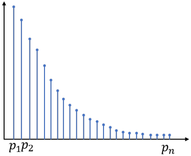

Low Entropy

EC (Entropy Constrained)-PEG Algorithm
For each VN v_j
Expand Tanner Graph in a BFS fashion
If $\exists$ CNs not connected to v_j
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Else *New cycles created*

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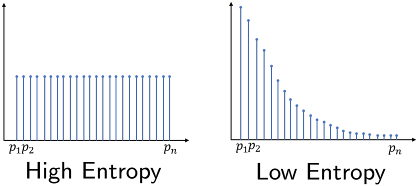

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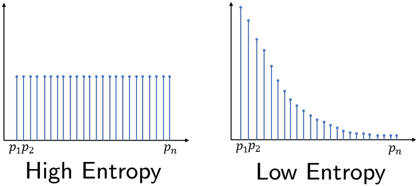

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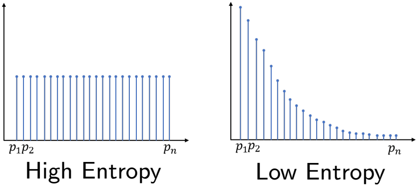

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- Update cycle distribution

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EC-PEG Algorithm

- Whenever a new edge, that creates cycles, is added to the Tanner Graph, we update the cycle counts of each VN

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VNs $(v_1, v_2, \ldots, v_n)$

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VNs $(v_1, v_2, \ldots, v_n)$

- $\lambda_i^g :=$ No. of cycles of length g that v_i is a part of, $g = 4, 6, 8$

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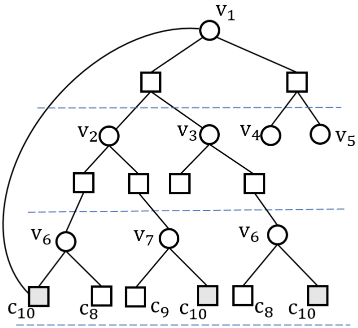

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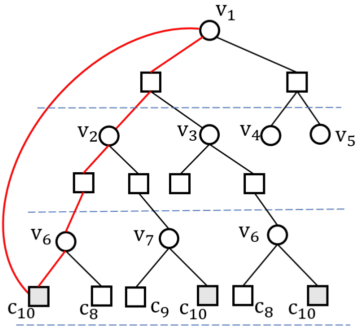

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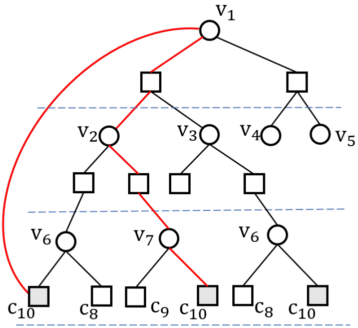

VNs $(v_1, v_2, \dots, v_n)$

- $\lambda_i^g :=$ No. of cycles of length g that v_i is a part of, $g = 4, 6, 8$

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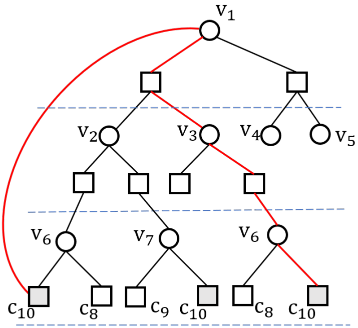

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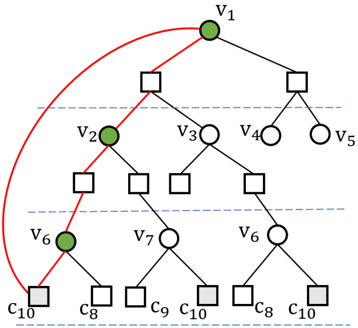

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- λ_i^g := No. of cycles of length g that v_i is a part of, $g = 4, 6, 8$
- $$\lambda_1^6 = \lambda_1^6 + 1$$
  $$\lambda_2^6 = \lambda_2^6 + 1$$
  $$\lambda_6^6 = \lambda_6^6 + 1$$

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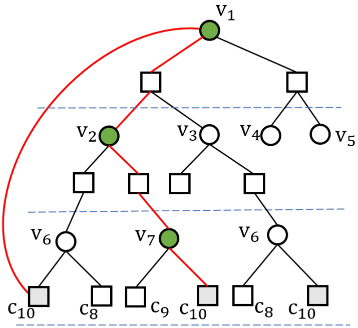

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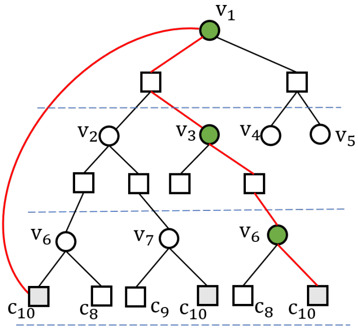

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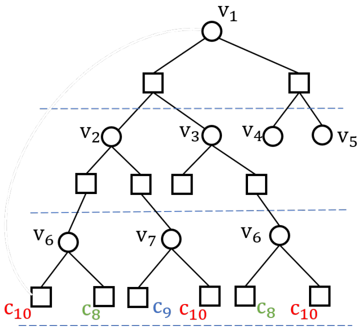
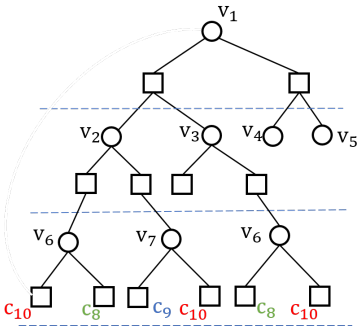
EC-PEG Algorithm: CN Selection Procedure



Candidate CNs : c_8, c_9, c_{10}

EC-PEG Algorithm: CN Selection Procedure



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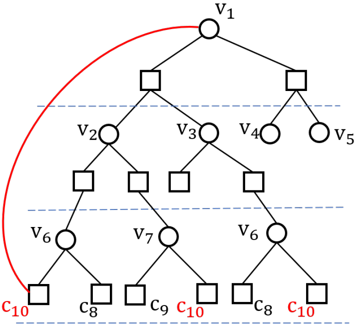
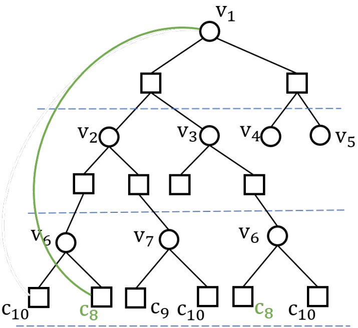
EC-PEG Algorithm: CN Selection Procedure



Candidate CNs : c_8, c_9, c_{10}

- For each CN candidate, calculate the resultant VN cycle counts
- $(\lambda_1^4, \ldots, \lambda_n^4), (\lambda_1^6, \ldots, \lambda_n^6), (\lambda_1^8, \ldots, \lambda_n^8)$

EC-PEG Algorithm: CN Selection Procedure



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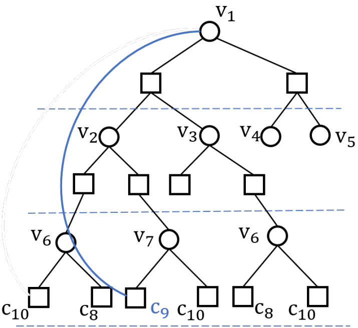

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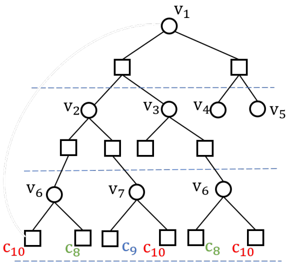

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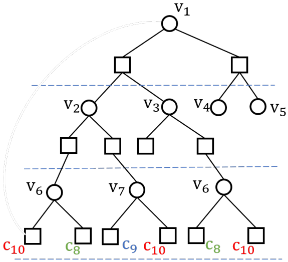

- $(\lambda_1^4, \ldots, \lambda_n^4), (\lambda_1^6, \ldots, \lambda_n^6), (\lambda_1^8, \ldots, \lambda_n^8)$
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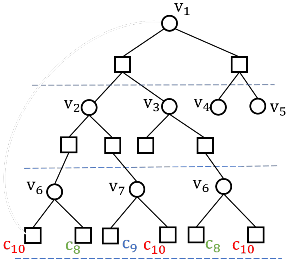
$$\underbrace{(\lambda_1^g, \ldots, \lambda_n^g)}_{\text{cycle counts}}$$

EC-PEG algorithm: CN selection Procedure



- $(\lambda_1^4, \dots, \lambda_n^4), (\lambda_1^6, \dots, \lambda_n^6), (\lambda_1^8, \dots, \lambda_n^8)$
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$$\underbrace{(\lambda_1^g, \dots, \lambda_n^g)}_{\text{cycle counts}} \rightarrow \underbrace{\left(\frac{\lambda_1^g}{\sum_{i=1}^n \lambda_i^g}, \dots, \frac{\lambda_n^g}{\sum_{i=1}^n \lambda_i^g}\right) := \alpha^g}_{\text{normalized counts}}$$

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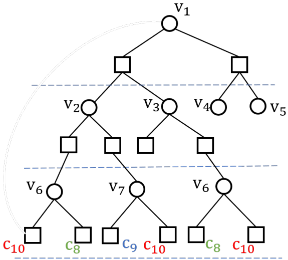

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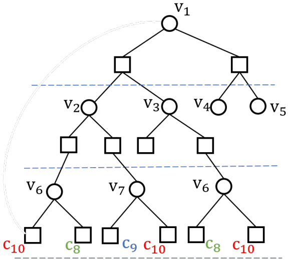

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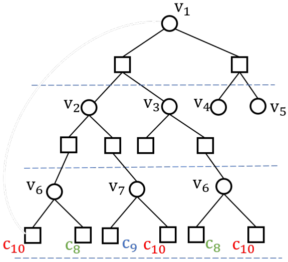

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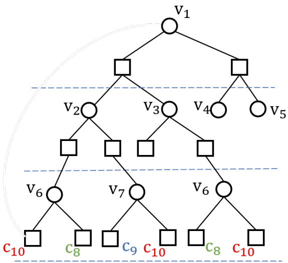

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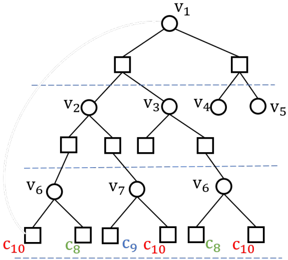

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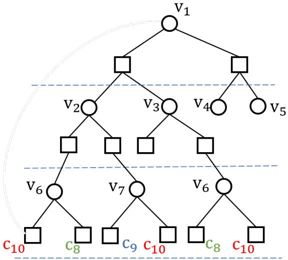
CN selection procedure:

EC-PEG algorithm: CN selection Procedure



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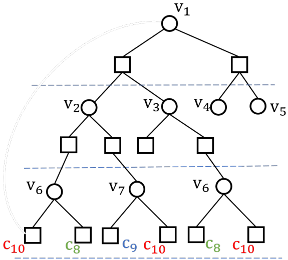
CN selection procedure:
Select CN that results in minimum $\mathcal{H}(\frac{\alpha^4+\alpha^6+\alpha^8}{3})$

EC-PEG algorithm: CN selection Procedure



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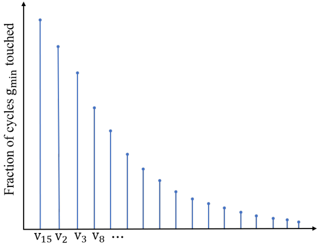
CN selection procedure:
Select CN that results in minimum $\mathcal{H}(\frac{\alpha^4+\alpha^6+\alpha^8}{3})$

Note:

- Minimizing the entropy of joint cycle counts ensures that all cycle distributions are concentrated towards the same set of VNs

Sampling Strategy

- Our sampling strategy greedily samples VNs that are part of a large number of cycles



$g =$ smallest cycle length in Tanner Graph $\mathcal{G}$
While sample set size $< s$
- $v =$ VN that is part of largest no. of cycles of length g in $\mathcal{G}$
- sample set $=$ sample set $\cup$ v
- remove v and all incident edges from $\mathcal{G}$

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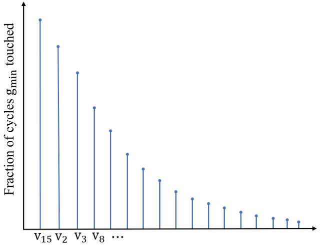

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- sample set $=$ sample set $\cup$ v
- remove v and all incident edges from $\mathcal{G}$

If $\nexists$ cycles of length g in $\mathcal{G}$
- $g = g + 2$

Simulation Results

- Code parameters: Code length $= 100$, VN degree $= 4$, Rate $= \frac{1}{2}$, girth $= 6$.

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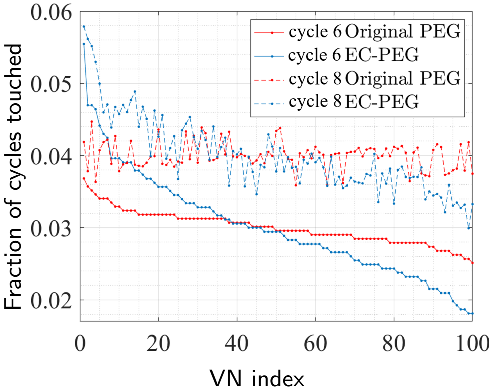

- VN indices arranged in decreasing order of cycle 6 fractions

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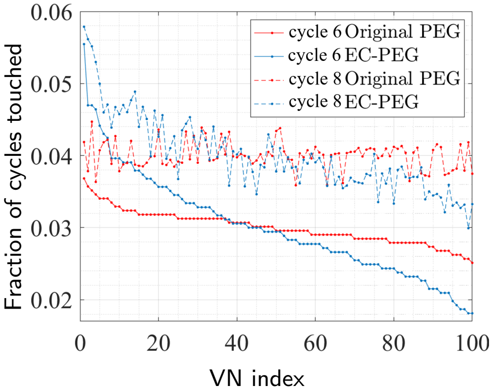

- VN indices arranged in decreasing order of cycle 6 fractions
- Cycle 6 and cycle 8 concentrated towards same set of VNs

Simulation Results

Fraction of SSs of size 11, 12 touched by different VNs

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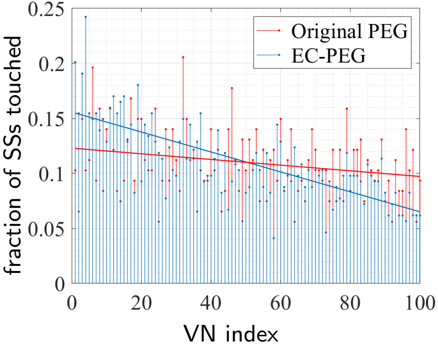
Fraction of SSs of size 11, 12 touched by different VNs

SSs of size 11



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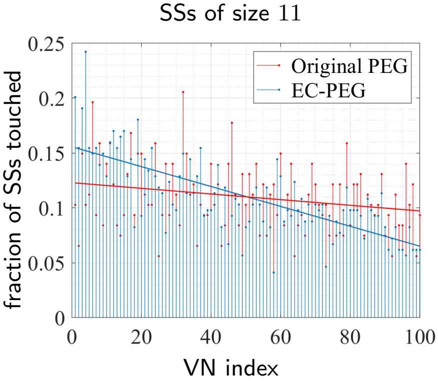
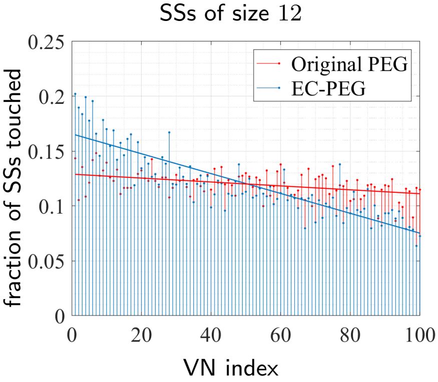
Simulation Results

Fraction of SSs of size 11, 12 touched by different VNs

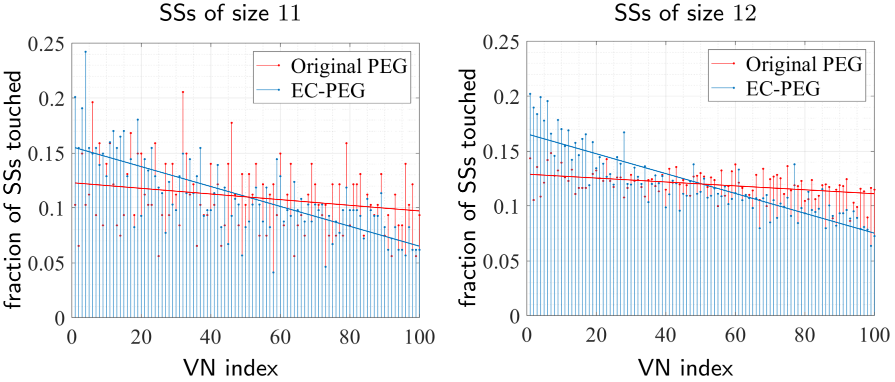




- VN indices arranged in decreasing order of cycle 6 fractions

Simulation Results

Fraction of SSs of size 11, 12 touched by different VNs



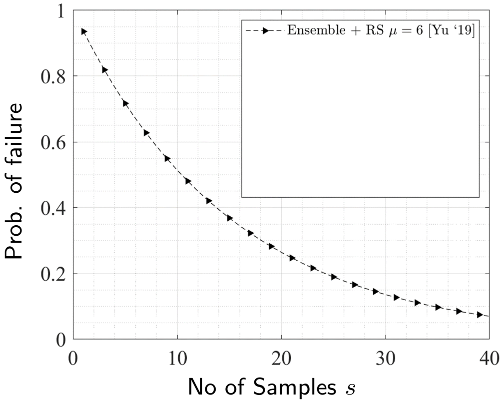
- VN indices arranged in decreasing order of cycle 6 fractions
- SSs are concentrated towards the same set of VNs as the cycles

Simulation Results

Probability of failure for a stopping set of size μ

Simulation Results

Probability of failure for a stopping set of size μ RS: Random Sampling



Simulation Results

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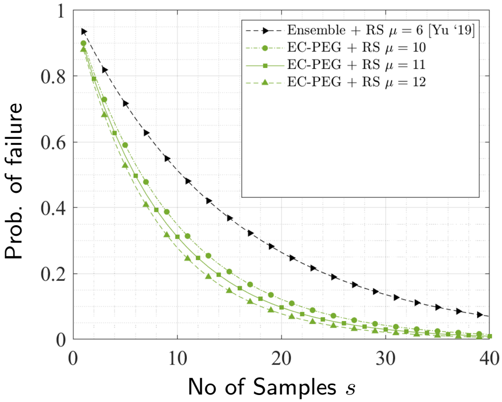
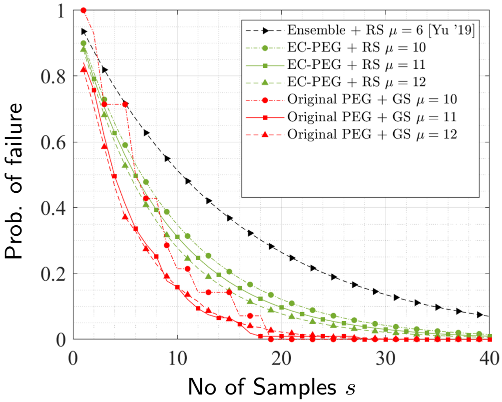


Simulation Results

Probability of failure for a stopping set of size μ

RS: Random Sampling
GS: Greedy Sampling



Simulation Results

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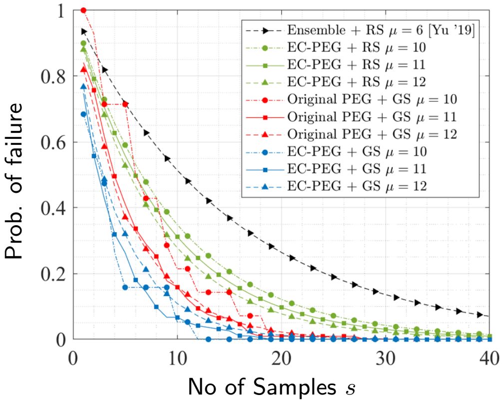

Simulation Results

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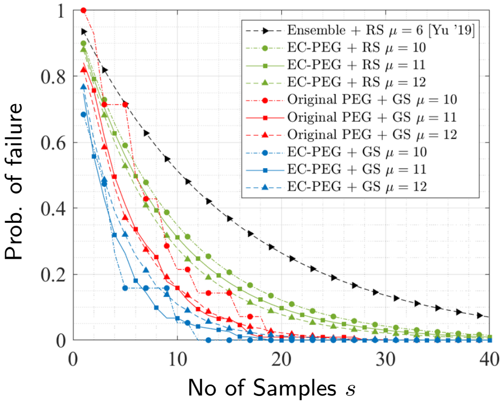

- Concentrated LDPC codes with Greedy sampling improve the probability of failure

Incorrect Coding Proof Size

- Depends on the maximum check node degree

Rate	Code length	VN degree	Ensemble [Yu '19]	PEG	EC-PEG
$\frac{1}{2}$	100	4	16	9	11
	200	4	16	9	15
$\frac{1}{4}$	100	4	8	7	10
	200	4	8	6	9

Table: Maximum CN degree for different codes.

Incorrect Coding Proof Size

- Depends on the maximum check node degree

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- Concentrated LDPC codes do not sacrifice on the incorrect coding proof size

Conclusion and Ongoing Work

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  - Coupled with a greedy sampling strategy, concentrated LDPC codes reduce the probability of light node failure compared to earlier approaches
- Ongoing work:
  - Improving security against stronger adversaries that can selectively pick a stopping set that has a lower probability of being sampled to hide

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