On the Sum-capacity of Compound MAC Models with Distributed CSI and Unknown Fading Statistics

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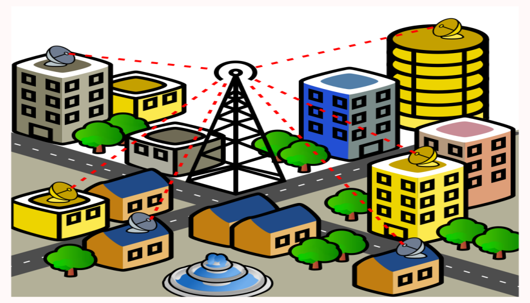
Motivation



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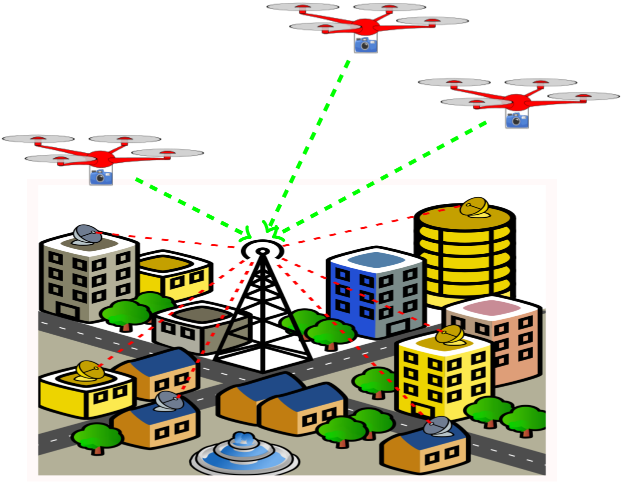

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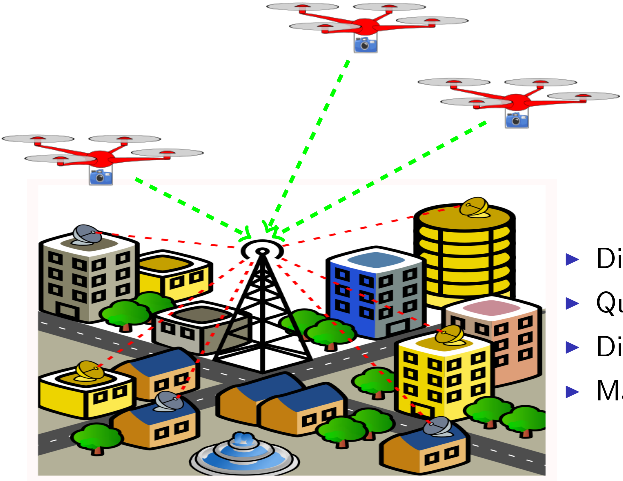


- Distributed Nodes
- Quasi-static Fading
- Different terrains
- Many fading CDFs

Compound MAC Model

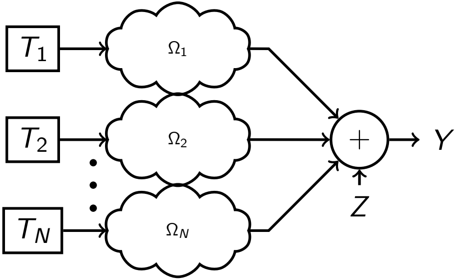


Figure: Distributed compound fading MAC model

$$Y = \sum_{i=1}^{N} H_i X_i + Z$$

$$P(H_i \leq h) = Q_i(h), Q_i \in \Omega_i.$$

Distributed Compound Adaptive Sum-Capacity

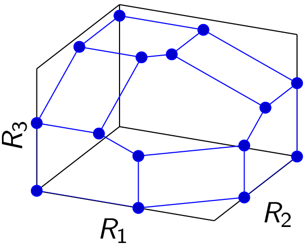


Figure: No-Outage Condition

Distributed Compound Adaptive Sum-Capacity

Optimization

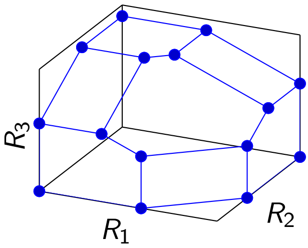
$$\max_{R_i(\cdot),P_i(\cdot)} \quad \min_{Q_1,\cdots,Q_N} \mathbb{E}\sum_{i=1}^{N} R_i(H_i)$$

subject to $\forall S \subseteq \{1,\cdots,N\}$,

$$\sum_{i\in S} R_i(H_i) \le \frac{1}{2}\log(1+\sum_{i=1}^{N} H_i^2 P_i(H_i))$$

and for $1 \le i \le N$,

$$\mathbb{E}P_i(H_i) \le P_i^{avg}.$$



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Feasibility:Mid-point Strategy

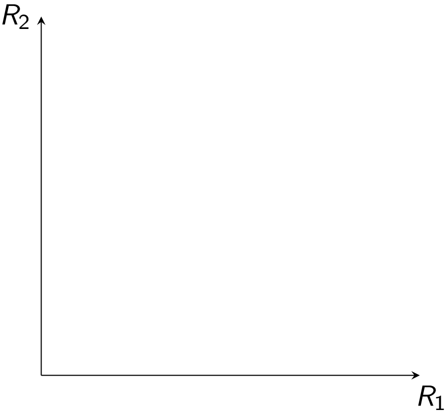

Figure: The users 1 and 2 construct the inner and outer symmetric regions respectively, and chooses A [1].

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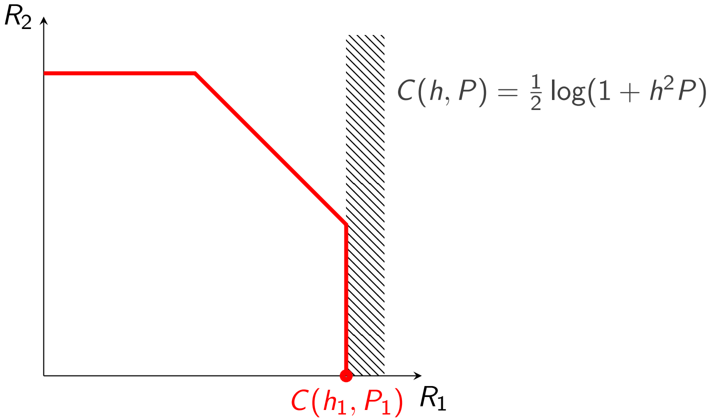

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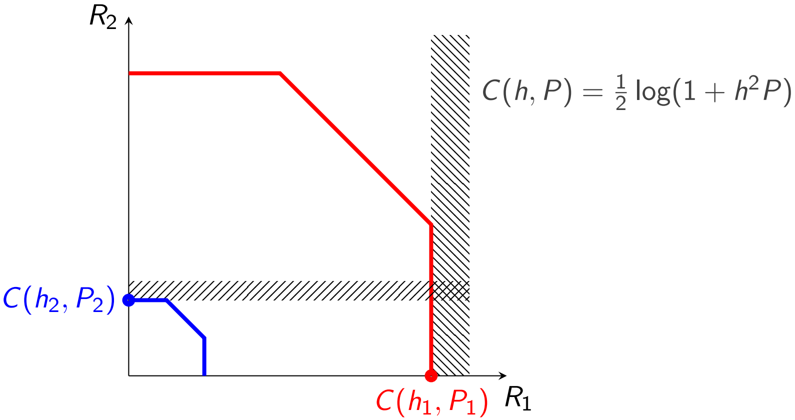

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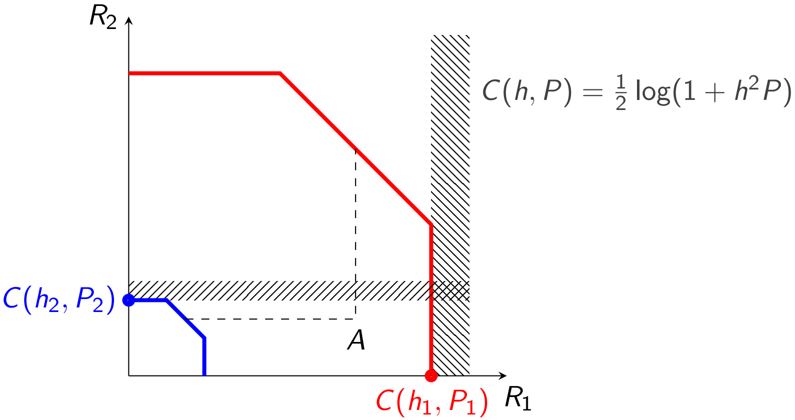

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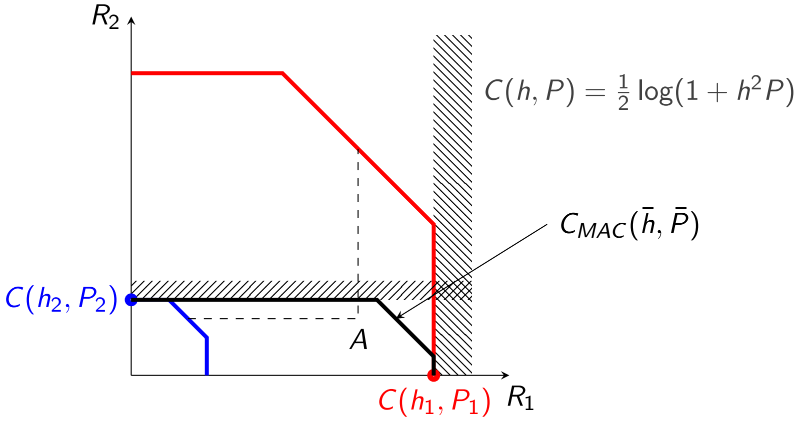

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Compound MAC with Identical Set of Distributions

Theorem

For identical set of fading distributions, i.e. $\Omega_1 =, \cdots, = \Omega_N = \Omega$,

$$\mathcal{C}_{sum} = \max_{P_1(\cdot), \cdots, P_N(\cdot)} \min_{w \in \Omega} \mathbb{E}\left[\frac{1}{2}\log(1 + H^2 \sum_{i=1}^{N} P_i(H))\right].$$

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Proof.

For any set of allocations $P_1(\cdot), \cdots, P_N(\cdot)$, and $Q \in \Omega$

$$\min_{Q_1,\cdots,Q_N} \mathbb{E}_{Q_1,\cdots,Q_N} \sum_{i=1}^{N} R_i(H_i) \overset{(a)}{\leq} \mathbb{E}_{Q,Q,\cdots,Q} \sum_{i=1}^{N} R_i(H)$$

$$\overset{(b)}{\leq} \mathbb{E}_Q \frac{1}{2}\log(1 + \sum_{i=1}^{N} |H|^2 P_i(H)).$$

□

Achievable Scheme

Taking $P_s(H) \triangleq P_1(H) + \cdots + P_N(H)$, consider the rate allocation

$$R_i(H_i) = \frac{P_i^{avg}}{\sum_{i=1}^{N} P_i^{avg}} \log(1 + H_i^2 P_s(H_i)).$$

The resulting sum-rate is $\mathbb{E}_{Q_1,\cdots,Q_N} \triangleq \mathbb{E}_{Q_1,\cdots,Q_N} \sum_{i=1}^{N} R_i(H_i)$.

Lemma

For a two user MAC,

$$\mathbb{E}_{Q,Q} \geq \mathbb{E}_{Q,Q'} \Rightarrow \mathbb{E}_{Q',Q'} \leq \mathbb{E}_{Q,Q'}.$$

Iterating the argument, the minimum indeed happens for some common distribution across users.

Single User Power Control

$$\mathcal{C}_{sum} = \max_{P_s(\cdot)} \min_{w \in \Omega} \mathbb{E}\left[\frac{1}{2}\log(1 + H^2 P_s(H))\right],$$

is a point to point problem over a compound channel.

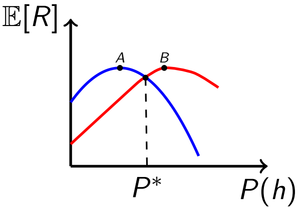

Figure: Optimal Power Control Solutions

Single User Power Control

Define the sub-problem:

$$
\begin{aligned}
J_i^* = \max_{P(h)} & \mathbb{E}_{Q_i} \log(1 + H^2 P(H)) \\
\text{s.t. } & \mathbb{E}_{Q_j}[P(H)] \le P^{avg}, 1 \le j \le K, \\
& \mathbb{E}_{Q_i}[\log(1 + H^2 P(H))] \le \mathbb{E}_{Q_j}[\log(1 + H^2 P(H))], \\
& \qquad 1 \le j \le K, j \ne i.
\end{aligned}
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$$\begin{aligned} L = &\mathbb{E}_{Q_i}[\log(1 + H^2 P(H))] - \sum_{i=1}^{K} \lambda_i [\mathbb{E}_{Q_i}(P(H)) - P^{avg}] \\ &- \sum_{j=1, j\ne i}^{K} \mu_j [\mathbb{E}_{Q_i} \log(1 + H^2 P(H)) - \mathbb{E}_{Q_j} \log(1 + H^2 P(H))] \end{aligned}$$

$$\lambda_k, \mu_k \ge 0$$

Algorithm for Single User Power Control

Input: Ω **Output:** $P^*(h)$
Initialize $P^*(h) = P^{avg}$.
For all $i \in \{1, \cdots, K\}$
Solve the sub-problem J_i^* and find the optimal $P_i(h)$.
endfor

$$P^*(h) = \underset{P_i(h), 1 \le i \le K}{\text{argmax}} \ \mathbb{E}_{Q_i}[\log(1 + |H|^2 P_i(H))].$$

Single User Power control Solution

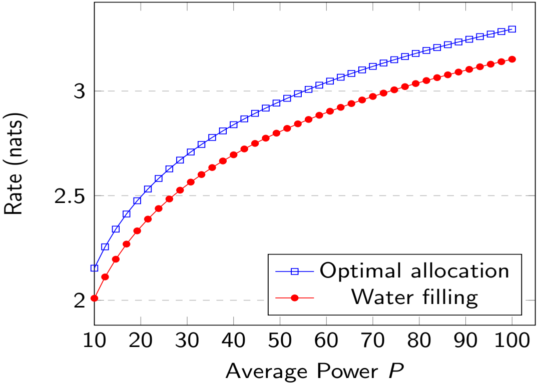

Figure: Single user capacity in a compound fading channel

$$\Omega = \{Q_1, Q_2\}, \text{ with } Q_1 = \mathrm{U}\{0, \cdots, 3\} \text{ and } Q_2 = \mathrm{U}\{0, \cdots, 4\}.$$

Conclusion

- Single user power control problem in a compound channel solves the compound MAC sum-capacity problem.
- An optimal rate allocation strategy which is completely distributed is identified for a symmetric compound MAC.

References

Y. Deshpande, S. R. B. Pillai, and B. K. Dey, "On the sum capacity of multiaccess block-fading channels with individual side information," *2011 IEEE Information Theory Workshop*, pp. 588–592, 2011.