Communication-Efficient LDPC Code Design for Data Availability Oracle in Side Blockchains

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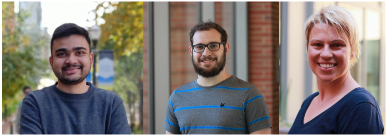
ITW 2021

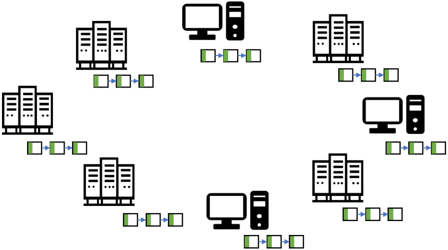


UCLA Samueli School of Engineering

Blockchain

- Distributed Ledger
- Decentralized trust platforms
- Application:
  - Finance and currency
  - Healthcare services
  - Supply chain management
  - Industrial IoT
  - e-voting

Central Problem: Poor Throughput and Latency



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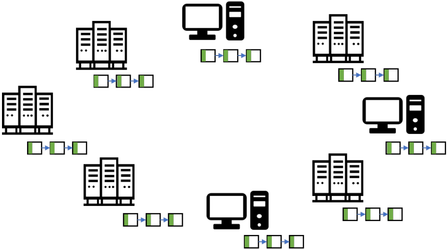

- Ledger of transaction blocks maintained by a network of nodes

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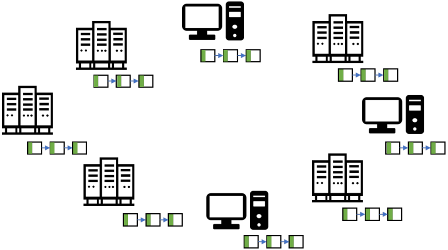

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Metrics:

- Transaction throughout: number of transactions processed in the system per second

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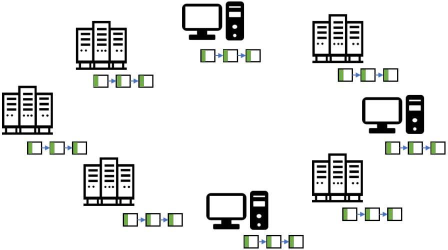

- Ledger of transaction blocks maintained by a network of nodes

Metrics:

- Transaction throughout: number of transactions processed in the system per second
- Confirmation latency: amount of time required for a transaction to be confirmed and deemed trustworthy

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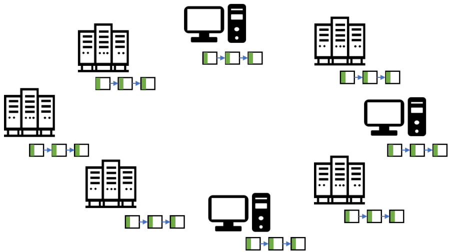

- Ledger of transaction blocks maintained by a network of nodes

	Transaction throughput	Confirmation Latency
Bitcoin Ethereum		

[Li '20]

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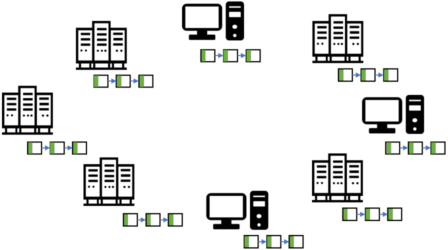

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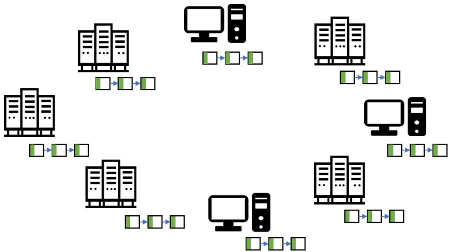

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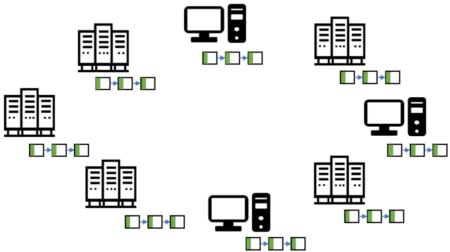

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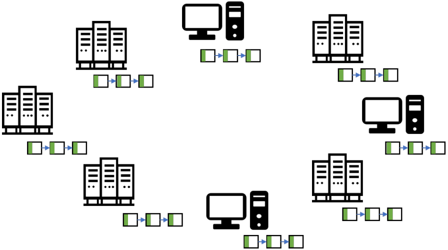

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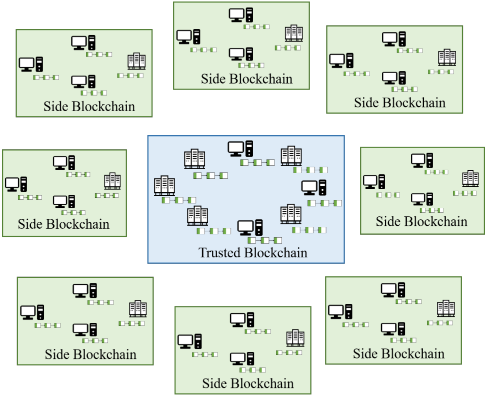
[Li '20]

Contrast: Visa processes more than 10,000 transactions/s[3]

[3] https://usa.visa.com

Solution: Running Side Blockchains



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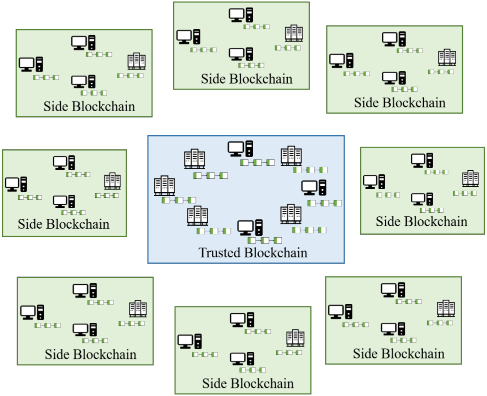

Side Blockchain:

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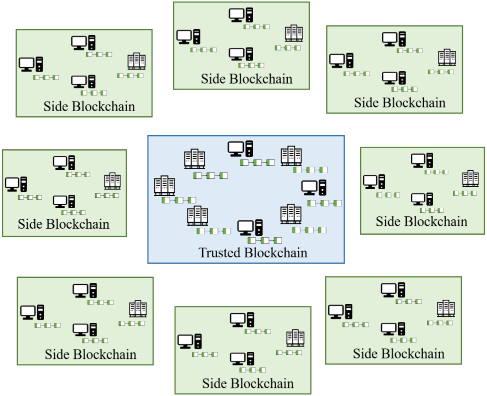
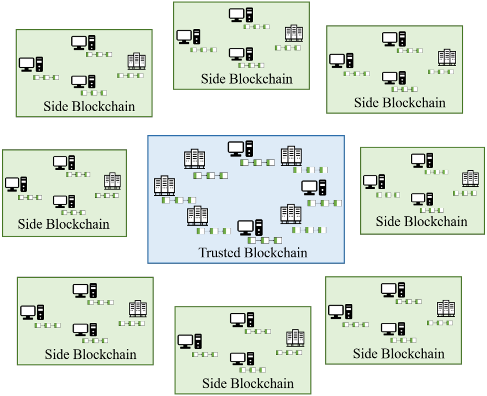


Side Blockchain:

- Smaller blockchain systems

Solution: Running Side Blockchains



Side Blockchain nodes:

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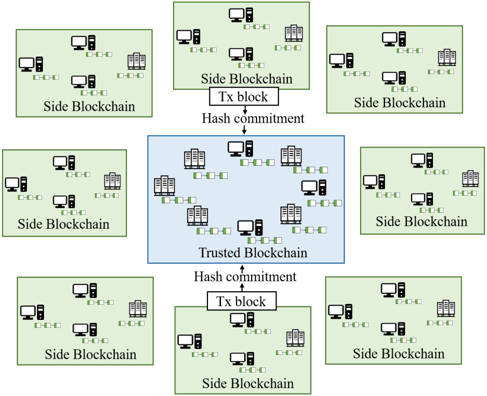

Side Blockchain nodes:

- Push hash commitment of their block to the trusted blockchain

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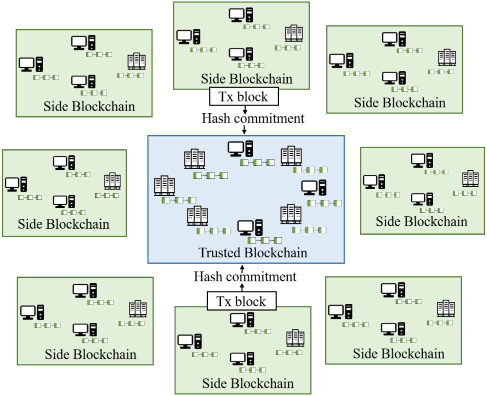

Side Blockchain nodes:

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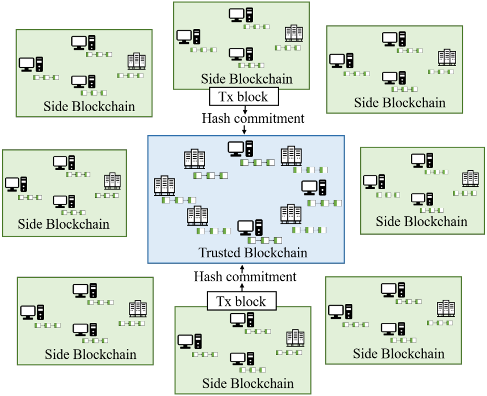

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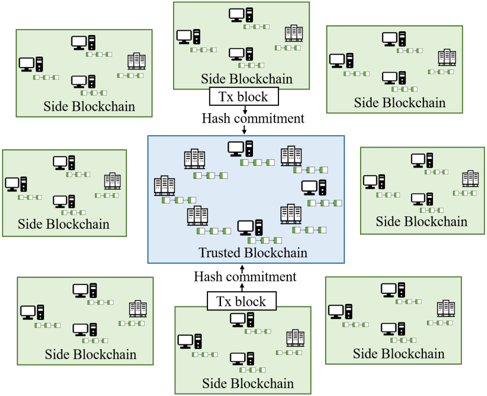

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- Only store the hash of the side blockchain

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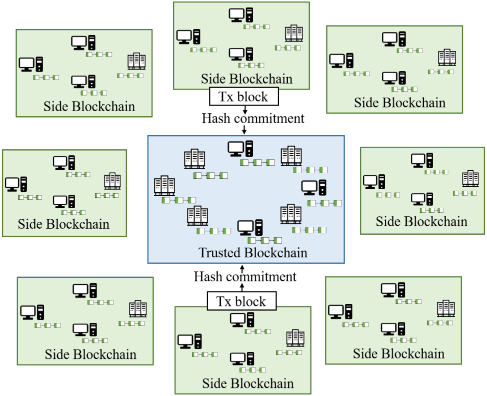

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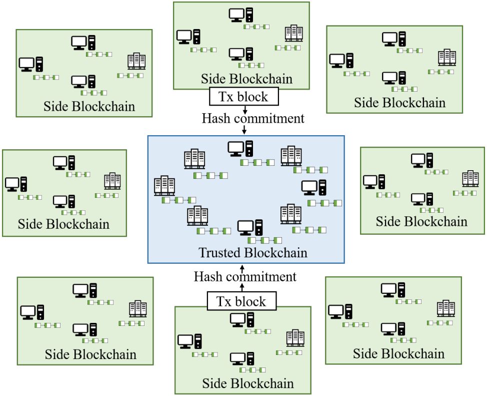

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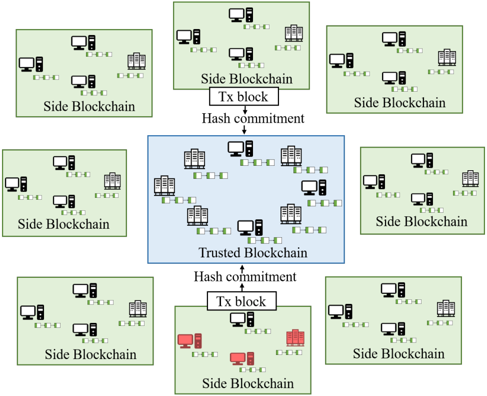

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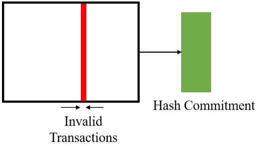
Issue: Side Blockchains with a majority of dishonest nodes are vulnerable to data availability attacks [Sheng '20]

Data Availability (DA) Attack in Side Blockchains

Adversary creates an invalid block



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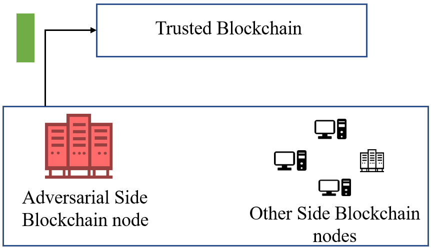

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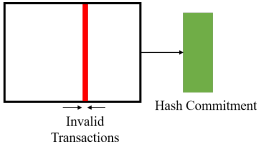

Adversarial Side Blockchain node:

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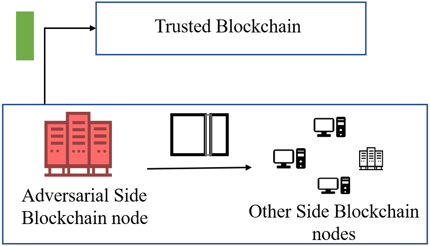

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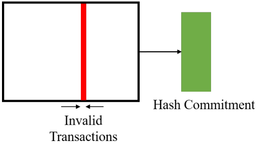

Adversarial Side Blockchain node:

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- Full block not available to other side blockchain nodes

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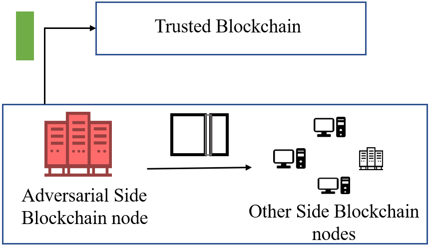

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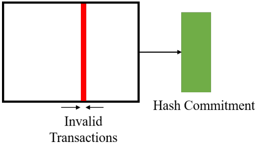

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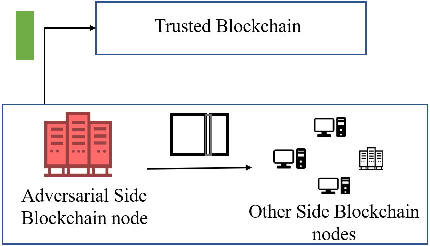

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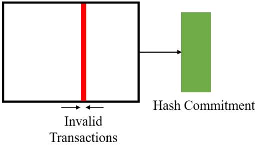

Note: DA attack cannot occur when side blockchains have a majority of honest nodes → majority vote on " is something missing?"

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Solution using a Data Availability Oracle

An oracle layer was introduced to ensure data availability [Sheng '20]

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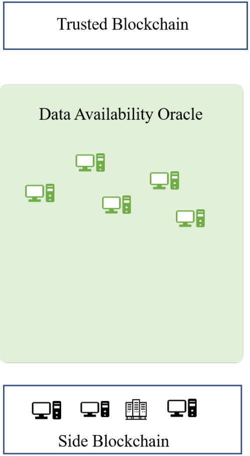

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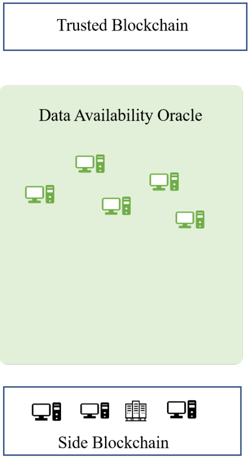

Oracle layer goal

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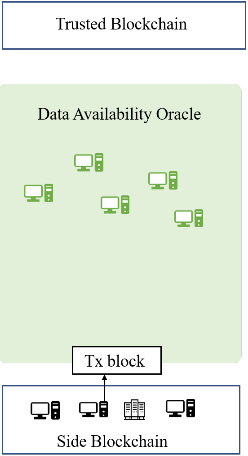

Oracle layer goal

- Accept a Tx block

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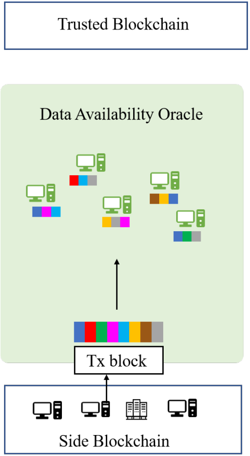

Oracle layer goal

- Accept a Tx block
- Collectively and efficiently store chunks of the Tx block (to guarantee availability)

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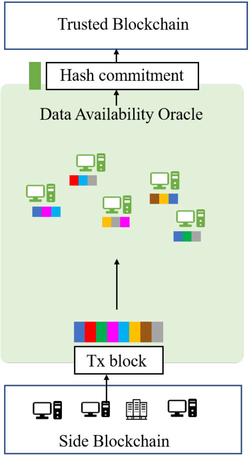

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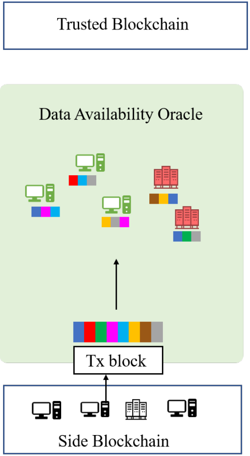

Oracle layer goal

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- Oracle nodes can be malicious (honest majority)

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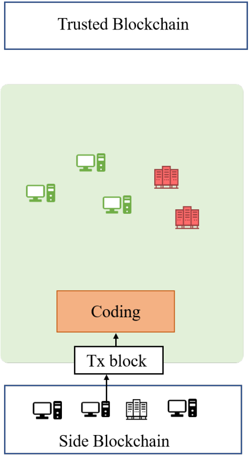
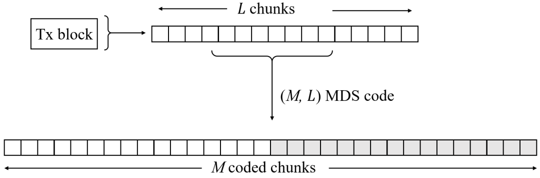


- Transaction block: chunked and coded



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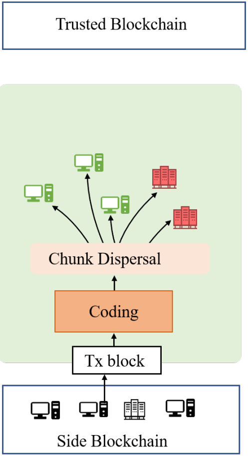
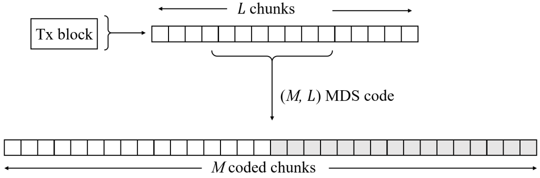


- Transaction block: chunked and coded
- Coded chunks dispersed among N oracle nodes



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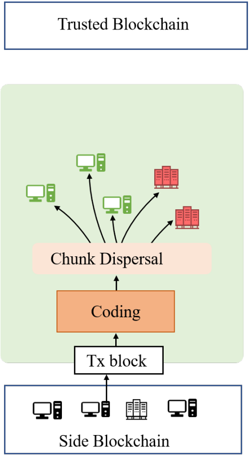

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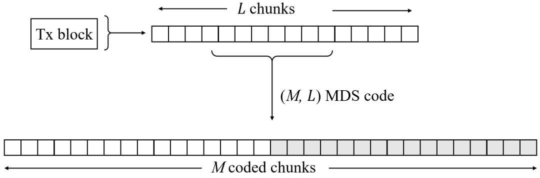

For MDS codes, iff at least L coded chunks are present among honest oracle layer nodes $\rightarrow$ block availability is guaranteed

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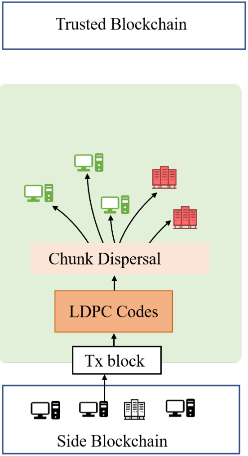

Low-Density Parity Check (LDPC) codes are used to code the Tx block

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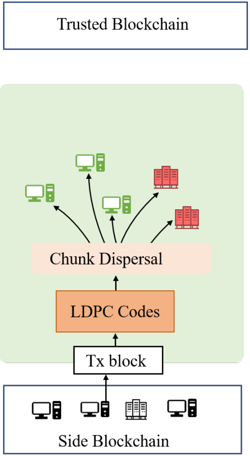

Low-Density Parity Check (LDPC) codes are used to code the Tx block

- Linear decoding complexity using a peeling decoder

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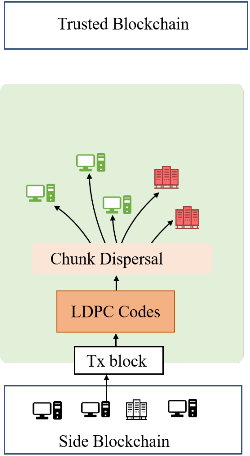

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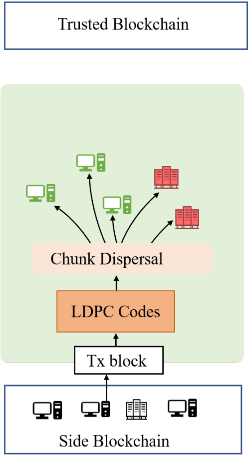

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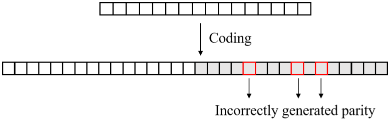

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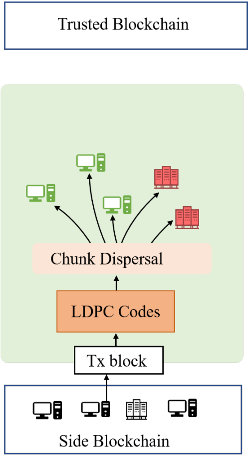

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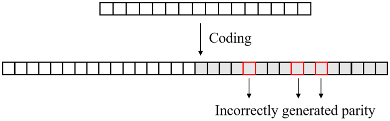

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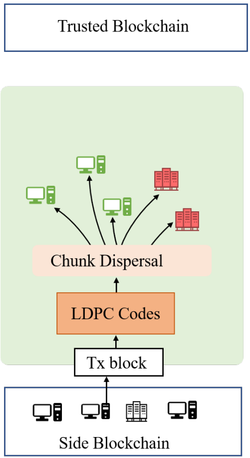

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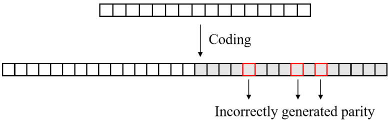

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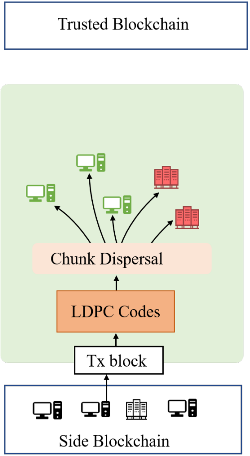

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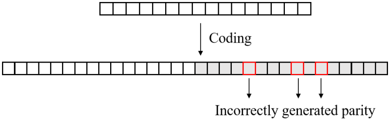

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- LDPC code have small incorrect coding proof size due to sparse parity check matrix

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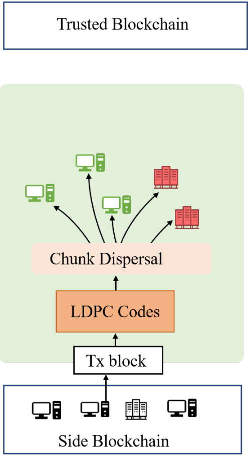

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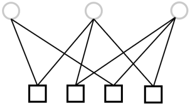
Issues with LDPC codes: small stopping sets



Solution using a Data Availability Oracle

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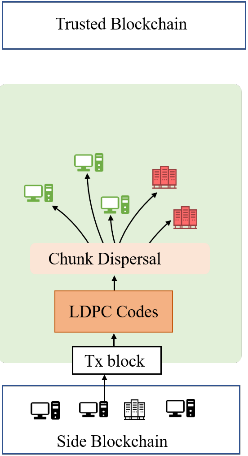

Low-Density Parity Check (LDPC) codes are used to code the Tx block

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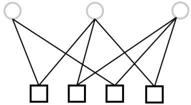

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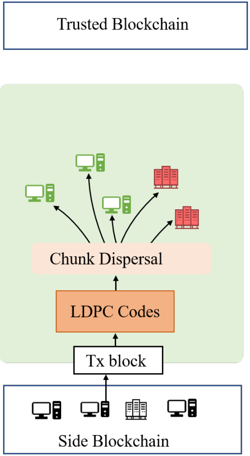

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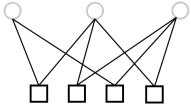

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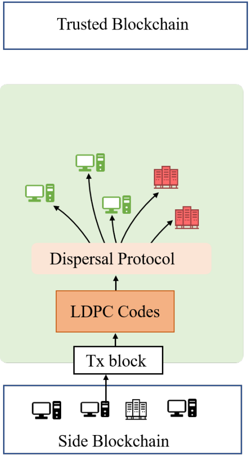

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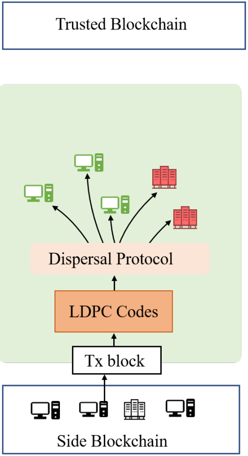

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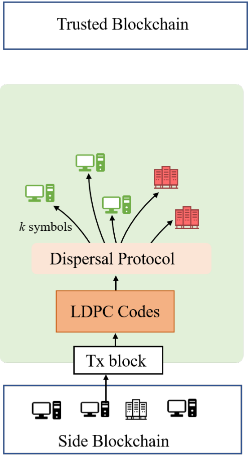

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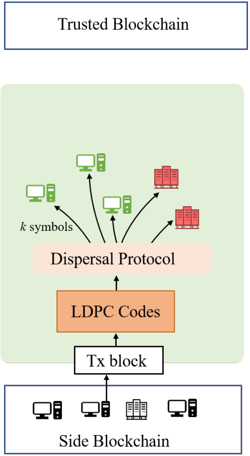

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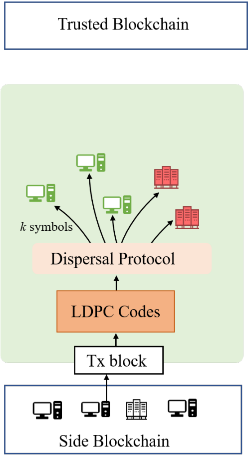

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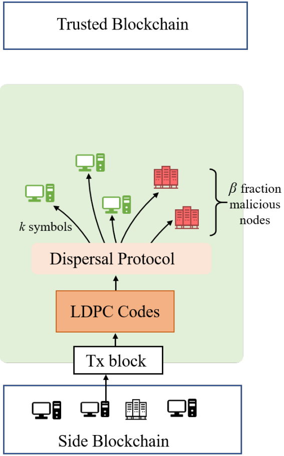

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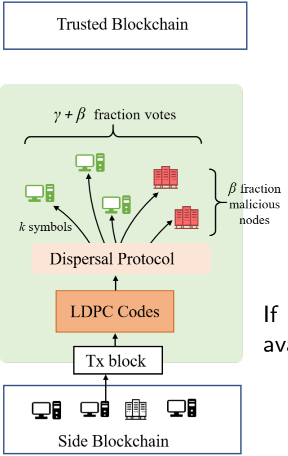

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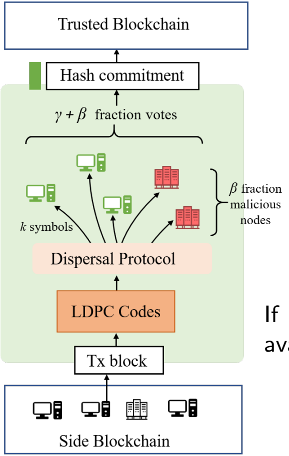

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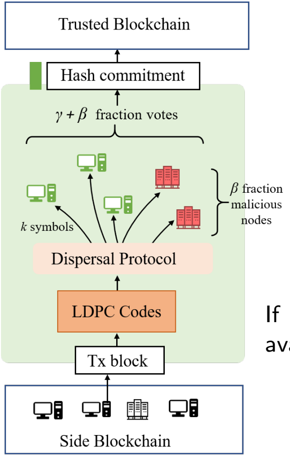

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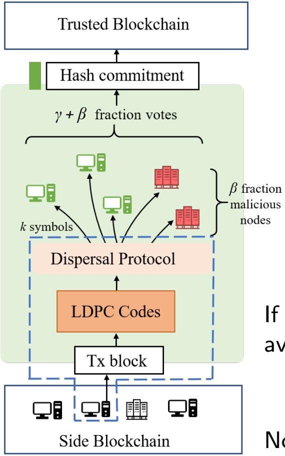

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Note: Side blockchain nodes perform LDPC encoding and dispersal

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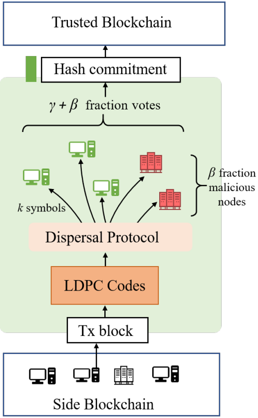

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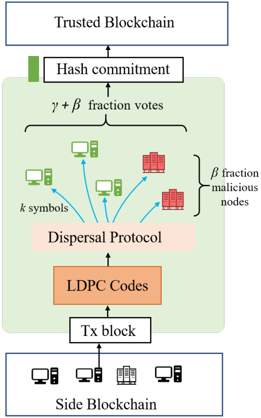

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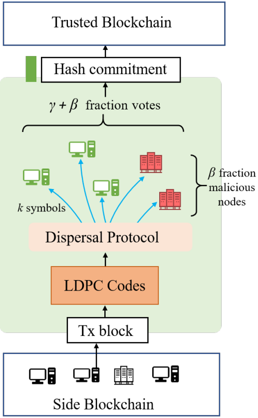

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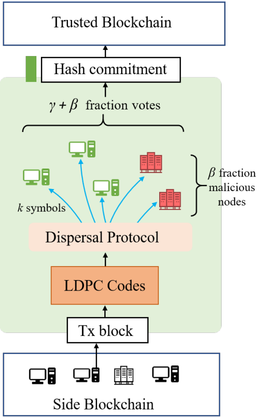

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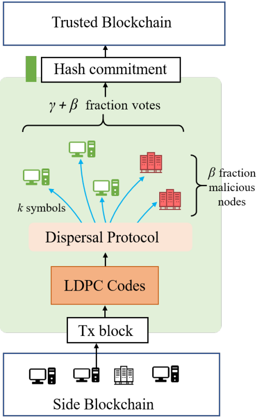

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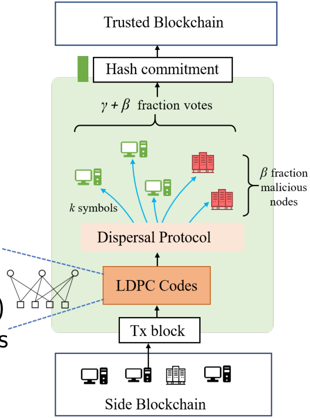

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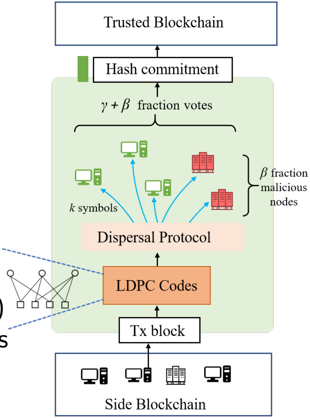

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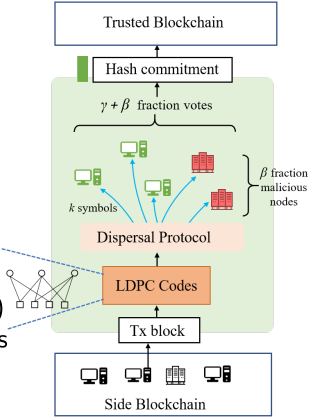

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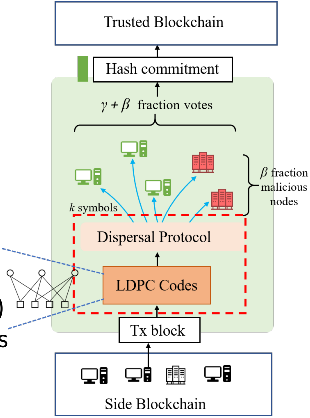

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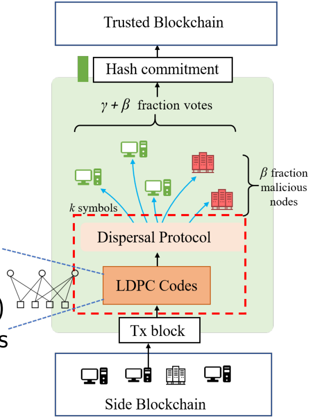

Our work: Design of specialized LDPC codes and a tailored dispersal protocol to significantly lower the communication cost.

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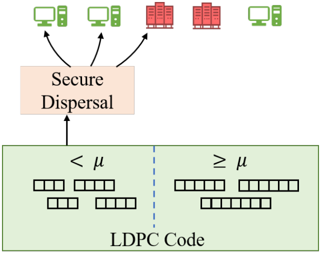

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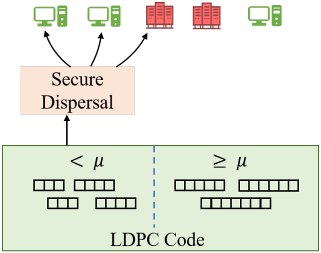

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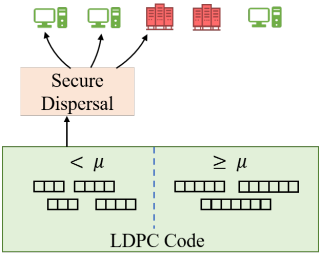

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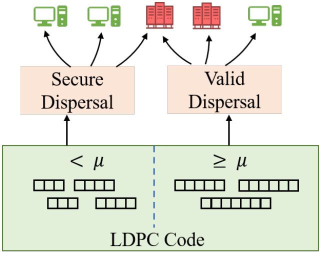

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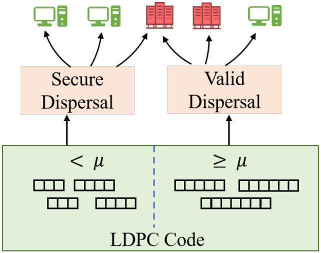

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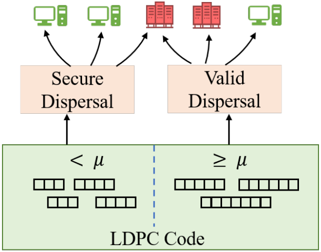

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> Code Design Strategy:
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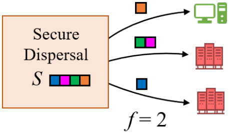

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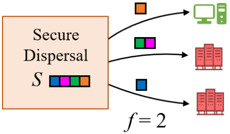

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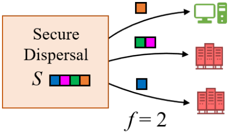

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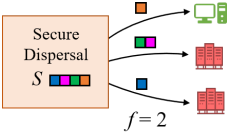

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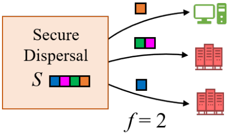

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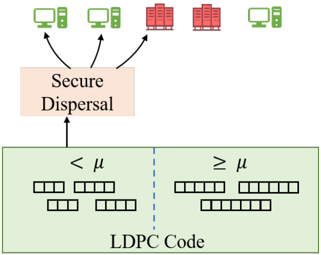

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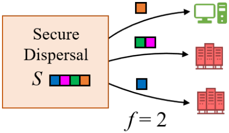

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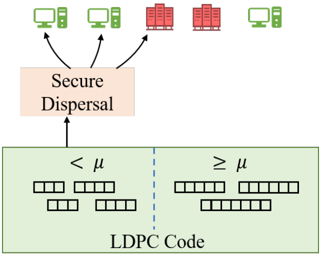

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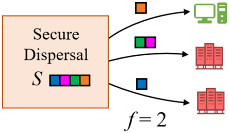

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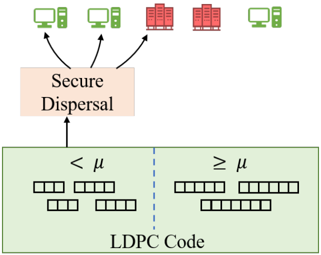

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- $\mathcal{V}$: set of VNs that *cover* all SSs in $\mathcal{S}$

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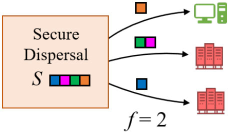

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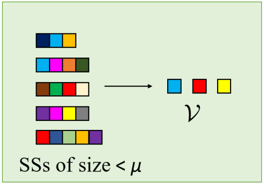

- $\mathcal{S} =$ All SSs of size $< \mu$
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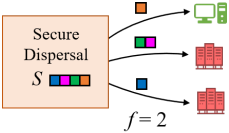
Secure Phase



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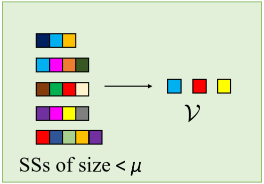

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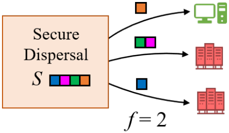

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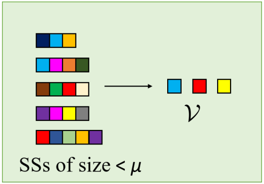

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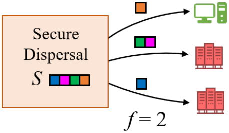

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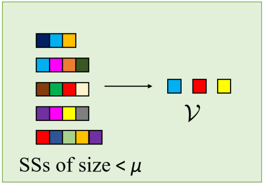

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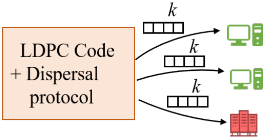

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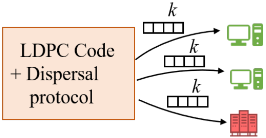

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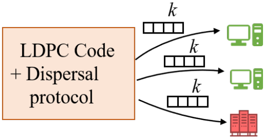

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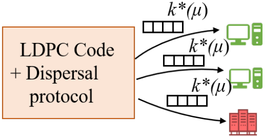

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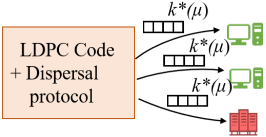

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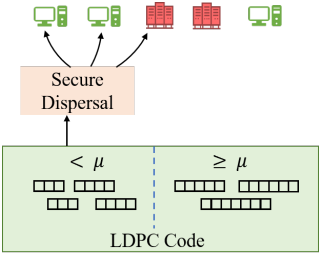

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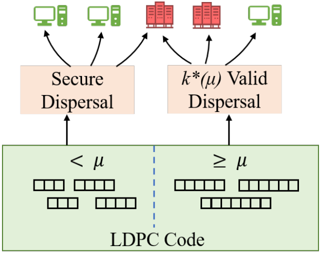

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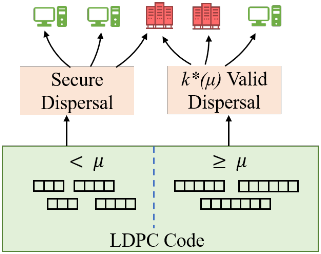

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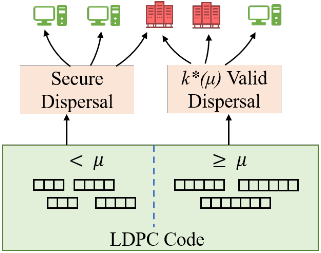

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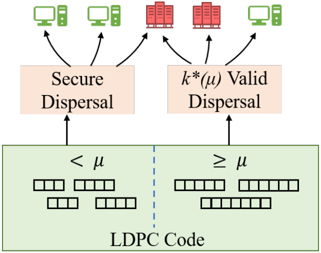

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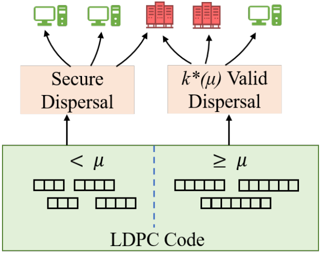

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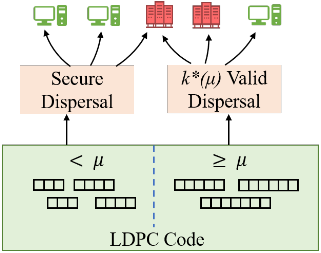

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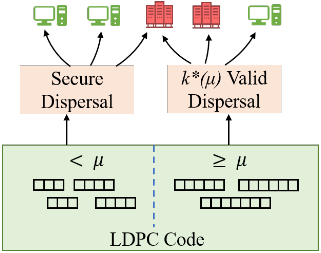

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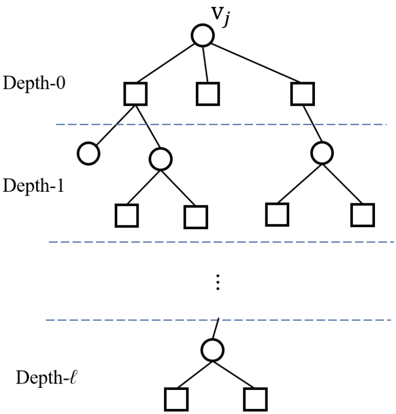
-Modify the PEG algorithm

PEG Algorithm

- Constructs a Tanner Graph in an edge by edge manner [Xiao '05]

PEG Algorithm



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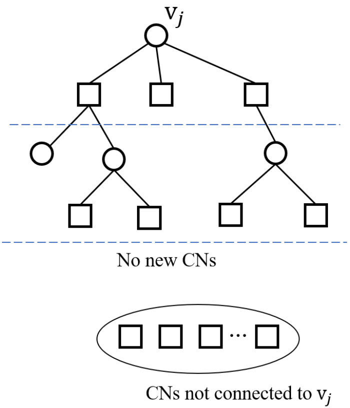

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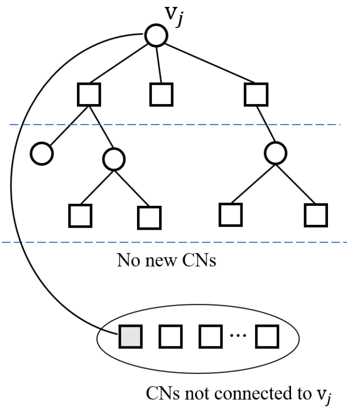

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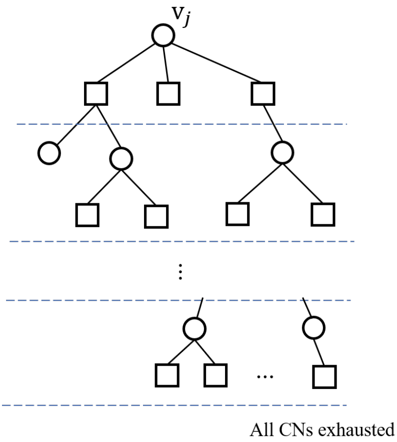

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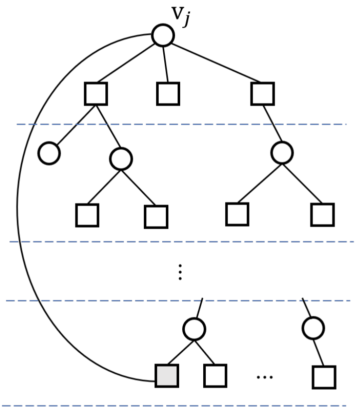

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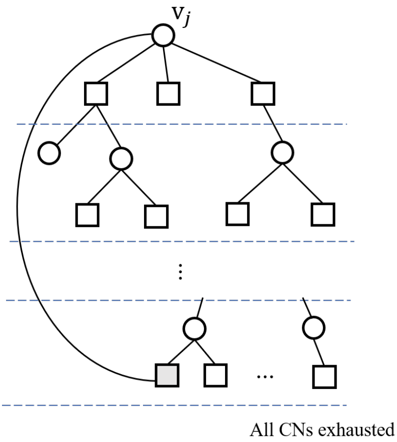

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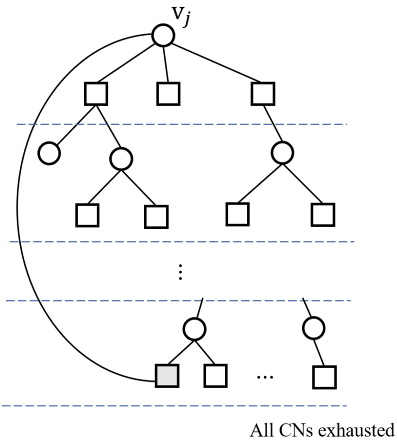

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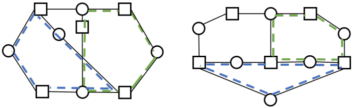
We modify the CN selection criteria in green to result in a low $|Greedy\text{-}Set(\mathcal{S})|$

Dispersal-Efficient (DE)-PEG Algorithm

- SSs are made up of cycles [Tian '03]



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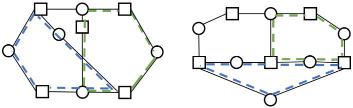

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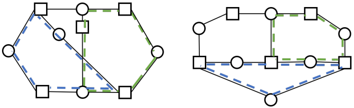

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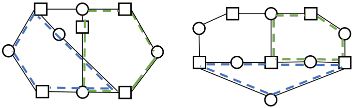

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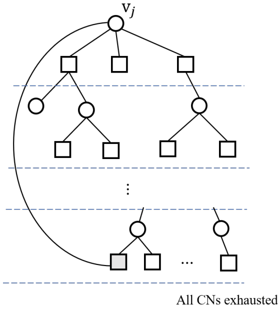

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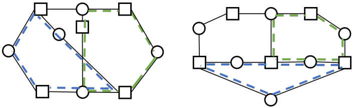

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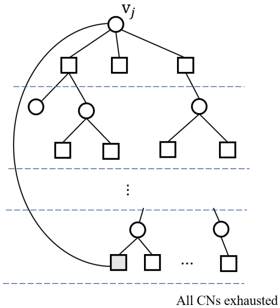

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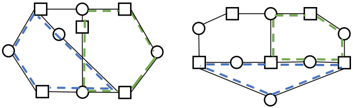

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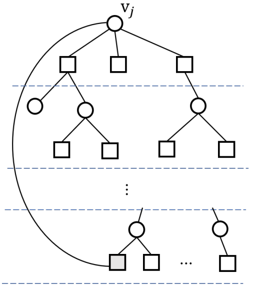

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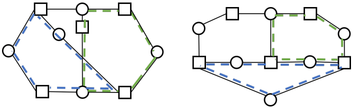
- Select one with minimum $|Greedy\text{-}Set(\mathcal{L})|$

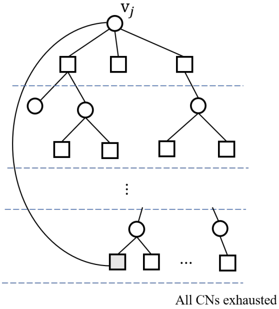
Issue: Using $\mathcal{L}$ that contains all cycles of length $\leq g$ does not reduce $|Greedy\text{-}Set(\mathcal{S})|$

Dispersal-Efficient (DE)-PEG Algorithm

- SSs are made up of cycles [Tian '03]
- Want to design LDPC codes with low $|Greedy\text{-}Set(\mathcal{S})|$, $\mathcal{S}$ = all SSs of size $< \mu$



- Design LDPC codes to reduce $|Greedy\text{-}Set(\mathcal{L})|$, $\mathcal{L}$ = List of cycles of length $\leq g$



DE-PEG Algorithm
For each VN v_j
Expand Tanner Graph in a BFS fashion
If $\exists$ CNs not connected to v_j
- Select a CN with min degree not connected to v_j

Else *(new cycles created)*
- Find CNs most distant to v_j
- Select CNs with minimum degree
- Select one with minimum $|Greedy\text{-}Set(\mathcal{L})|$

Solution:
Make $\mathcal{L}$ contain only low Extrinsic Message Degree (EMD) [Tian '04] cycles

Simulation Results: Communication Cost Reduction

System Parameters: $N = 9000$, $\beta = 0.49$, $M = 256$, Block size = 1MB, $p_{th} = 10^{-8}$, LDPC code rate = $\frac{1}{2}$, $\gamma = 1 - 2\beta$. All communication costs are in GB.

Simulation Results: Communication Cost Reduction

System Parameters: $N = 9000$, $\beta = 0.49$, $M = 256$, Block size = 1MB, $p_{th} = 10^{-8}$, LDPC code rate = $\frac{1}{2}$, $\gamma = 1 - 2\beta$. All communication costs are in GB.

- $|\mathcal{V}| = |Greedy\text{-}Set(\mathcal{S})|$ for $M = 256$, $\mathcal{S} =$ all SS of size $< \mu$

μ	$\lvert\mathcal{V}\rvert$	
	PEG	DE-PEG
17	0	0
18	1	0
19	3	1
20	7	4
21	14	13

Simulation Results: Communication Cost Reduction

System Parameters: $N = 9000$, $\beta = 0.49$, $M = 256$, Block size = 1MB, $p_{th} = 10^{-8}$, LDPC code rate = $\frac{1}{2}$, $\gamma = 1 - 2\beta$. All communication costs are in GB.

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- DE-PEG always results in lower $|\mathcal{V}|$ compared to PEG

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- $|\mathcal{V}| = |Greedy\text{-}Set(\mathcal{S})|$ for $M = 256$, $\mathcal{S}$ = all SS of size $< \mu$
- C^s: communication cost of secure phase of dispersal

Secure Phase

μ	$\lvert\mathcal{V}\rvert$		C^s	
	PEG	DE-PEG	PEG	DE-PEG
17	0	0	0	0
18	1	0	0.037	0
19	3	1	0.112	0.037
20	7	4	0.262	0.149
21	14	13	0.524	0.486

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- C^s: communication cost of secure phase of dispersal
- C^v: communication cost of valid phase of dispersal (each node gets $k^*(\mu)$ chunks)

Secure Phase | Valid Phase

μ	$\|\mathcal{V}\|$		C^s		C^v
	PEG	DE-PEG	PEG	DE-PEG	
17	0	0	0	0	5.116
18	1	0	0.037	0	4.887
19	3	1	0.112	0.037	4.658
20	7	4	0.262	0.149	4.428
21	14	13	0.524	0.486	4.276

- DE-PEG always results in lower $|\mathcal{V}|$ compared to PEG
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Secure Phase Valid Phase

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Simulation Results: Communication Cost Reduction

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- C^s: communication cost of secure phase of dispersal
- C^v: communication cost of valid phase of dispersal (each node gets $k^*(\mu)$ chunks)
- C^T: total communication cost = $\mathrm{C}^v + \mathrm{C}^s + \Delta$ (small additional overhead)

Secure Phase | Valid Phase

μ	$\|\mathcal{V}\|$ PEG	$\|\mathcal{V}\|$ DE-PEG	C^s PEG	C^s DE-PEG	C^v	C^T PEG	C^T DE-PEG
17	0	0	0	0	5.116	5.125	5.125
18	1	0	0.037	0	4.887	4.933	4.896
19	3	1	0.112	0.037	4.658	4.779	4.704
20	7	4	0.262	0.149	4.428	4.700	4.587
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Secure Phase — Valid Phase

μ	$\lvert\mathcal{V}\rvert$		C^s		C^v	C^T	
	PEG	DE-PEG	PEG	DE-PEG		PEG	DE-PEG
17	0	0	0	0	5.116	5.125	5.125
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- DE-PEG always results in lower $|\mathcal{V}|$ compared to PEG
- As μ is increased, C^s increases. C^s for DE-PEG $<$ C^s for PEG, C^v decreases.
- C^T is lowest for $\mu = 20$, lower for DE-PEG

Simulation Results: Communication Cost Reduction

μ	$\lvert\mathcal{V}\rvert$		C^s		C^v	C^T	
	PEG	DE-PEG	PEG	DE-PEG		PEG	DE-PEG
17	0	0	0	0	5.116	5.125	5.125
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- $M_{\min}$ for PEG LDPC code is 17.

Simulation Results: Communication Cost Reduction

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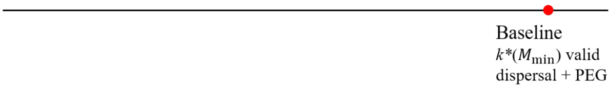

Simulation Results: Communication Cost Reduction

μ	$\lvert\mathcal{V}\rvert$		C^s		C^v	C^T	
	PEG	DE-PEG	PEG	DE-PEG		PEG	DE-PEG
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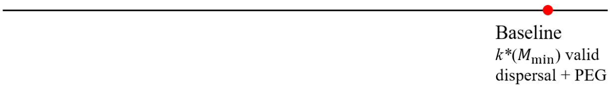

Simulation Results: Communication Cost Reduction

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	PEG	DE-PEG	PEG	DE-PEG		PEG	DE-PEG
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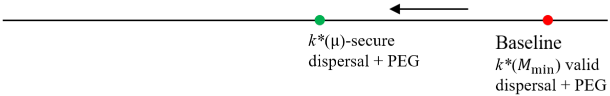
- $\mu = 17$ is considered as the baseline with $k^*(M_{\min})$ valid dispersal protocol (no secure phase)
- Using k^*-secure dispersal protocol with $\mu = 20$ reduces C^T from baseline:
  Reduction for PEG: 0.425GB



Simulation Results: Communication Cost Reduction

μ	$\lvert\mathcal{V}\rvert$ PEG	$\lvert\mathcal{V}\rvert$ DE-PEG	C^s PEG	C^s DE-PEG	C^v	C^T PEG	C^T DE-PEG
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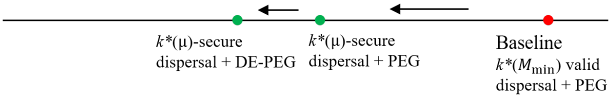

Simulation Results: Communication Cost Reduction

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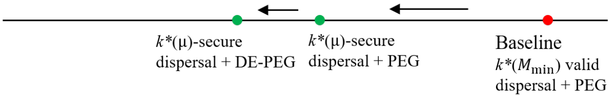
  Reduction for PEG: 0.425GB
  Reduction for DE-PEG: 0.528GB
- Lower bound on C^T for $\mu = 20$ is 4.438GB (assuming $\mathrm{C}^s = 0$)



Simulation Results: Communication Cost Reduction

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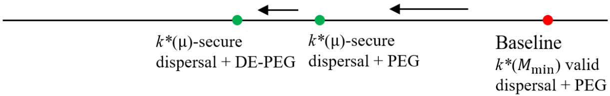
  $\rightarrow$ equivalent to designing codes with larger minimum SS size which is hard



Simulation Results: Communication Cost Reduction

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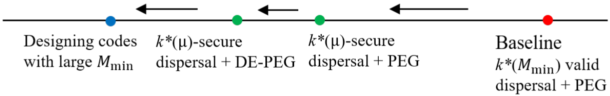
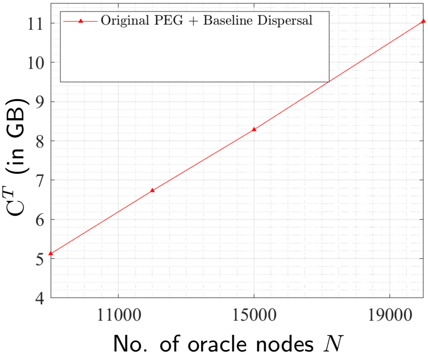


Simulation Results



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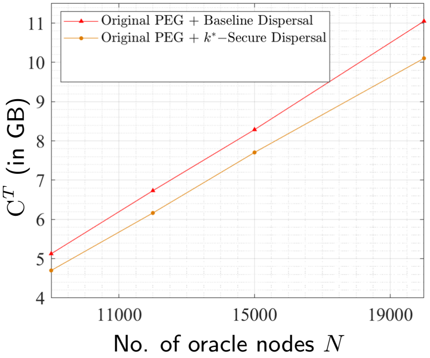
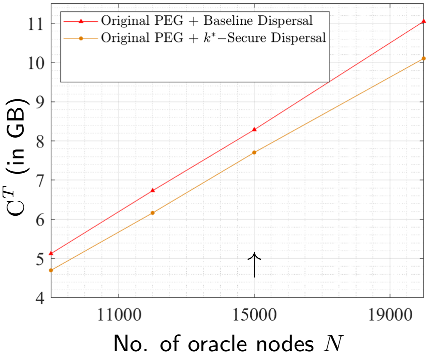


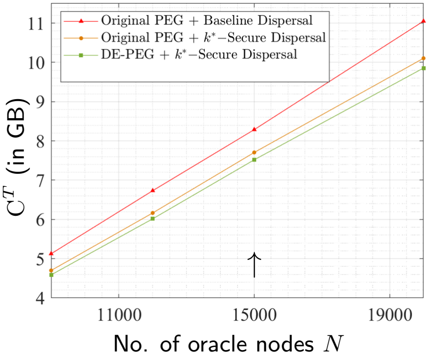
Simulation Results



At $N = 15000$

- Baseline $\xrightarrow{7\%\text{reduction}}$ PEG + k^*-secure dispersal protocol with $\mu = 20$

Simulation Results



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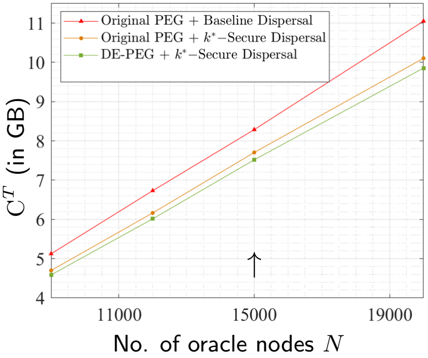
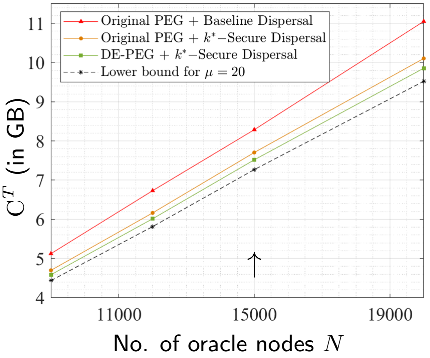


At $N = 15000$

- Baseline $\xrightarrow{7\%\text{reduction}}$ PEG + k^*-secure dispersal protocol with $\mu = 20$
- Baseline $\xrightarrow{9.3\%\text{reduction}}$ DE-PEG + k^*-secure dispersal protocol with $\mu = 20$

Simulation Results



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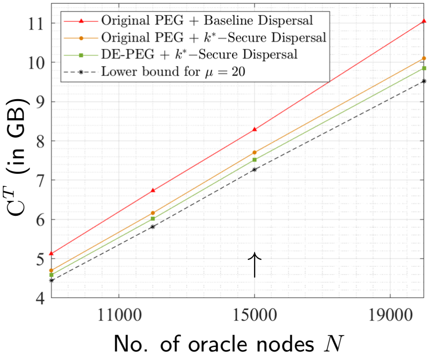
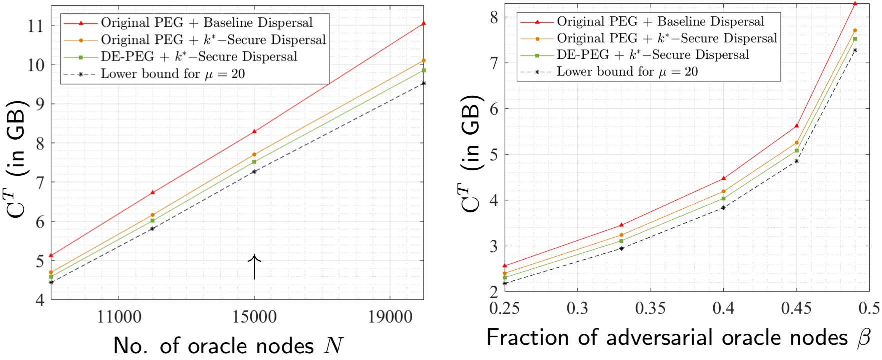
Simulation Results



At $N = 15000$

- Baseline $\xrightarrow{7\%\text{reduction}}$ PEG + k^*-secure dispersal protocol with $\mu = 20$
- Baseline $\xrightarrow{9.3\%\text{reduction}}$ DE-PEG + k^*-secure dispersal protocol with $\mu = 20$
- Baseline $\xrightarrow{13\%\text{reduction}}$ Lower bound for $\mu = 20$

Simulation Results



At $N = 15000$

- Baseline $\xrightarrow{7\%\text{reduction}}$ PEG + k^*-secure dispersal protocol with $\mu = 20$
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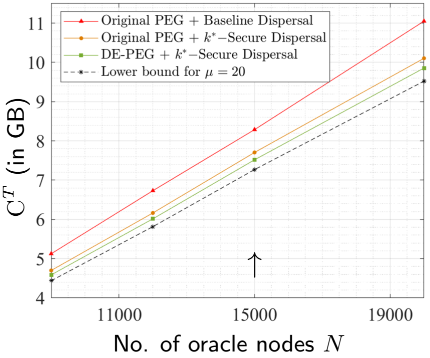
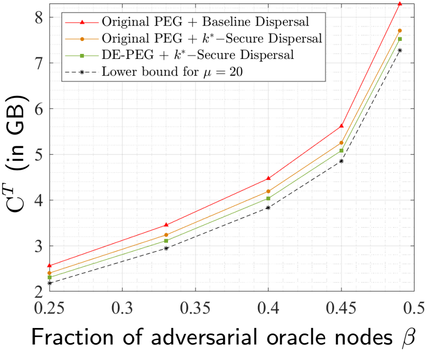
- Baseline $\xrightarrow{13\%\text{reduction}}$ Lower bound for $\mu = 20$

Simulation Results



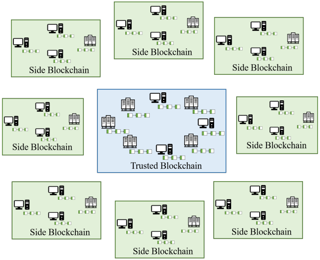


At $N = 15000$

- Baseline $\xrightarrow{7\%\text{reduction}}$ PEG + k^*-secure dispersal protocol with $\mu = 20$
- Baseline $\xrightarrow{9.3\%\text{reduction}}$ DE-PEG + k^*-secure dispersal protocol with $\mu = 20$
- Baseline $\xrightarrow{13\%\text{reduction}}$ Lower bound for $\mu = 20$
- Similar trends hold when C^T is plotted as a function of the adversary fraction β

Conclusion and Ongoing work

- Conclusion
  - Off-the-shelf LDPC codes, e.g. those designed for AWGN or BSC channels, may not be optimal for:
    ↳ Adversarial erasures with dispersal protocol



Conclusion and Ongoing work

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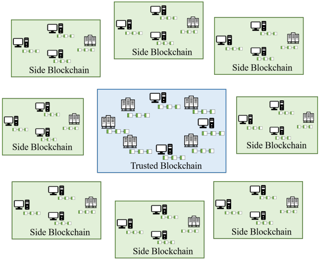

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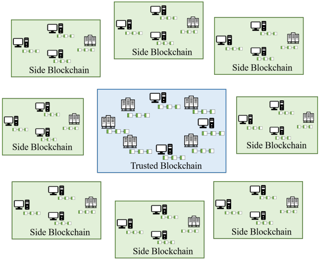

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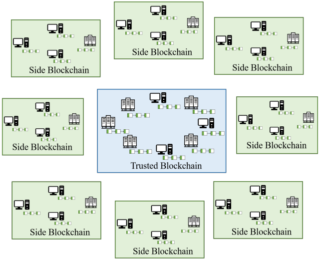

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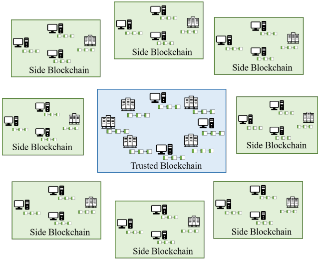

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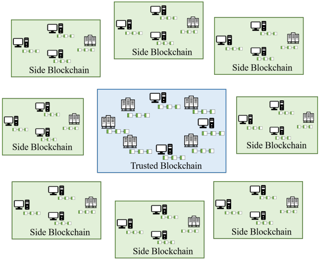

- Ongoing work
  - Considering other code families such as Polar codes for this application.

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